Committed Presentism:
Quantum Mechanics, Becoming, and Historical Truth

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Abstract

Committed Presentism is a single-history framework for quantum mechanics, becoming, and historical truth. Only the present exists: the past no longer exists, and the future does not. Within each domain of common origin, exactly one present is actual. A cosmic foliation fixes which distant events share it and the order of becoming. Becoming is primitive. The quantum state is nomic structure, not another concrete object, and the ontology selects neither collapse nor its absence. Definite outcomes are values of actual variables, and the future is open wherever the law admits alternatives.

Every realization must reproduce the Born statistics of recorded readouts. In quantum equilibrium at a fixed background, the unitary realization reproduces the observable marginals of quantum mechanics, and independent settings cannot signal. On finite configuration spaces, persisting records exhibit Born frequencies. The paper classifies the Born-preserving transition rates at a single present and bounds the setting dependence of admitted histories. It constructs effective models of matter and geometry in two restricted cosmological sectors. Realized closed causal curves or violations of the Born statistics would refute the framework.

For atomic transitions on chosen slices, past propositions are evaluated over the histories admitted into the present. A proposition is true if every such history satisfies it, false if none does, and otherwise has no truth value. Past propositions can lose determinacy, future ones can gain it, and determinate values never reverse. Propositions determinate at finitely many presents, each while it was actual, are jointly consistent. Custodial truth preserves historical distinctions, never the past itself. Under the constructed jump laws on finite configuration spaces with a constant Hamiltonian, past configurational truth perishes except across cuts no admitted transition crosses. The framework agrees with the block view of the same law wherever it assigns a value and differs exactly where admitted histories disagree.

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1 Introduction

Presentism holds that only the present exists. Taken as physics, it must answer three questions. Which distant events share our present, when the causal structure of relativity does not say? If one present is actual, what in it carries a definite outcome, and what is the quantum state? And if the past no longer exists, what makes a statement about it true? Committed Presentism (CP) is a single-history presentist ontology of quantum mechanics that answers all three.

Consider a galaxy in an expanding universe with a positive cosmological constant, so far away that no signal will ever pass between it and us. Its worldlines and ours run back to the same initial singularity. On CP that galaxy is real now, on the same terms as the Earth. It belongs to the one actual present whether or not its light ever arrives [Section 2.2, Remark 10]. A cosmic foliation fixes which of its events shares our present. CP posits this structure because causal structure alone does not supply it [Remark 13].

That is the foundation of the theory. Within each causal domain of common origin exactly one unfragmented present is actual. In a big-bang cosmology such a domain holds the events whose past runs back to the initial singularity. Within it there is one fact about what is real, the same from every vantage point. Global becoming then needs a fact about which events are co-present, and the cosmic foliation supplies it. The present is represented on one spacelike hypersurface of the foliation, and the order of these hypersurfaces is the order in which the present becomes. To the present itself CP adds one primitive posit, intrinsic becoming. The present is not one three-dimensional section of a completed four-dimensional whole; it becomes. The name Committed Presentism marks this commitment: the present actually becomes.

One physical reality is actual and concrete. The quantum state is lawlike structure: a mathematical input to the one physical dynamics that governs the present's change. A physical dynamics meeting the axioms supplies the law of the quantum state. This paper develops a unitary dynamics, in which the state obeys its adopted unitary or constraint equations without collapse. Stochastic collapse with a mass-density primitive ontology [3] is a candidate dynamics. Definite outcomes are features of the one actual physical reality. The variables that have definite values in it carry them, and their law admits alternative courses of change. Bell-type jump processes [9] are possible forms of that law; the axioms do not include them. The axiom of quantum consistency requires every such dynamics to assign probabilities to the readouts recorded in those definite variables. The present quantum state fixes these probabilities through a Born rule. A dynamics is refuted when its probabilities disagree with those measured. In quantum equilibrium the unitary realization reproduces the observable marginals of quantum mechanics at a fixed background, and independent settings cannot signal.

What has happened was actual when it happened. For atomic transition laws, the posit of historical truth classifies propositions about the past by histories. A proposition about the past is true now when it holds on every history that the law admits and that ends in the present. It is false when it holds on none, and otherwise it has no truth value. These histories test what the present and the law fix, and none of them is an existing past. A past truth can then lose its truth value where admitted histories merge, and it never becomes false. A proposition about the future is judged the same way, on the courses of change the law admits from the present. It only gains a truth value [Section 8].

What is claimed. The claim is compatibility regime by regime. Each result collected in Section 9 holds on the axioms together with the physics adopted in its regime. Every realization must supply what Section 7 states. In the Kantowski-Sachs and $\mathbb{T}^3$ minisuperspace sectors, that section constructs one law of quantum geometry and matter that meets the stochastic clause. Novelty lies in the formulation: an explicit axiomatic statement, univocity at the existence root, and the treatment of historical truth. Definite values carried beside a state that never collapses have modal antecedents [5]. The reading of the quantum state as lawlike structure has

an antecedent in Bohmian mechanics [26]. Grounding past truth in the present and the laws, with a sentence true when every course they allow agrees, has an antecedent in Markosian [46]. A past left open by indeterministic laws has antecedents as well [24, 44]. The truth-value gap is a supervaluation over histories [62, 63], taken here over the histories that the present and the law admit. Presentism on an objective foliation, which Bohmian mechanics accommodates, has been argued to be an open position [31].

Structure of the paper. The paper opens with the one present, univocity, the global now, and the incompatibility that the cosmic foliation answers [Section 2]. Intrinsic becoming and the completeness of the present follow [Section 3], then the one dynamics and the cosmic foliation with its causal placement law [Section 4]. Measurement is placed within becoming in Section 5. The remaining sections state gravitational compatibility [Section 6], the physical realizations [Section 7], historical truth [Section 8], and the established results with their scope [Section 9]. The appendix [Appendix A] carries the proofs the main text does not give inline.

2 One Present

Only the present exists. The past no longer exists, the future does not exist, and the present is one: not a patchwork of local presents, and the same from every vantage point. Two axioms say this. One unfragmented present is actual within each causal domain of common origin [A1], and within a domain there is one fact about what is real [A2]. Together they give CP's criterion of reality, and univocity makes the present a global now, shared by every real event of the domain. Causal structure alone does not say which distant events those are, so CP posits the cosmic foliation [A6] to place the present in the geometry.

2.1 The one present

Axiom A1 (One present [20, 23, 45])**.** *Within each causal domain of common origin exactly one unfragmented present is actual. No past or future configuration coexists with it.*

The axiom makes three claims. First, the present is one. Even where the quantum state evolves without collapse [A5], the alternatives it describes are not further actual presents or worlds: one physical reality is actual, and the state is lawlike structure belonging to it [Definition 6]. Second, the present is unfragmented. No part of it has a separate standing, and no part is more or less real than another [Remark 10]. Third, nothing else exists beside it. No earlier configuration persists alongside the present and no later one awaits it [Definitions 8 and 9].

Definition 1 (Realization)**.** A *realization* of the theory is a physical dynamics offered to meet its axioms. It specifies a law of the quantum state, the *actual physical variables* (the variables of the present that it takes to have definite values) with a law for them, the placement data of the cosmic foliation, and the distribution of the actual physical variables where its data are given. These are its *realization inputs*, which the axioms leave free [Section 7.1]. Where the actual physical variables are a point of a configuration space, that point is the *actual configuration*; in other realizations they are field or geometric data of the present. A realization is *admissible* when it meets every axiom, and its *realized geometry* is the spacetime geometry that represents the advance of the present under it [Remark 4]. In a *unitary realization* the quantum state obeys its adopted unitary or constraint equations without collapse, and the actual configuration follows a stochastic law whose rates read the present quantum state; *the unitary realization* is the class of these that this paper develops, with Bell-type jump processes as its developed member [Section 5.2]. A *joint realization* governs matter, quantum fields, and gravity by one coupled dynamics. A single law or posit is realized by a construction that meets it, as Bell-type jump processes realize the stochastic law of the actual physical variables [A5].

A domain comprises the events that the realized geometry, a representation [Remark 4], places downstream of one connected piece of its past causal boundary [A2]. In a big-bang cosmology that piece is the initial singularity. Where the geometry has several such origins, each domain carries its own present. At a fixed background the geometry fixes which domains each present event belongs to; for the coupled dynamics, the realization must fix it [Section 6].

2.2 Causal domain univocity

Axiom A2 (Univocity within a causal domain)**.** *Within one domain [Definition 2] there is one fact about what is real, never a family of facts indexed to vantage point or frame (thin univocity). Membership in the actual present has a single extension across the domain's events, independent of vantage, so co-membership in the present is fixed by that one fact (the co-membership clause). Common origin alone determines which domains an event belongs to. Neither a shared foliation nor instantaneous simultaneity determines domain membership. A cosmic event horizon can therefore sever contact without splitting the domain. Where domains overlap, their presents agree on the common events.*

Definition 2 (Causal domain)**.** A domain is a region of common origin: the set of events through which passes some past-inextendible timelike curve whose past end lies in one connected component of the past causal boundary of the realized geometry [Remark 4]. That component is the domain's origin, and the boundary is the one built from terminal indecomposable future sets [33], taken with the chronological topology [32, Defs. 3.10, 3.23]. The component alone determines the set, so no choice of congruence enters.

CP asserts no synchronized present across causally disconnected domains. Within a domain, the cosmic foliation fixes which event of each worldline belongs to the present [A6, Remark 12].

Definition 3 (Slice and cosmic parameter)**.** The cosmic foliation [A6] divides the realized geometry into a family of spatial *slices*, labelled by a continuous *cosmic parameter* τ that orders them. The slice labelled by τ is S_τ, and τ is its *slice parameter*. The *actual slice* is the one on which the present is represented. Slices and their parameters belong to the spacetime representation [Remark 4]: they place the present and order its advance, and are not further presents. A slice, unqualified, is a slice of the cosmic foliation.

Example. In an expanding spatially flat FLRW geometry that begins at an initial singularity, with a perfect fluid of positive density and $\Lambda > 0$, the expansion rate never falls below the asymptotic rate $h = c\sqrt{\Lambda/3}$. The scale factor then grows at least as e^{ht}, and a future-directed light ray from an event of the slice at proper time t meets only comoving worldlines within proper distance c/h on that slice. In the representation, no future-directed causal curve from an event of ours on the present slice meets the worldline of a comoving galaxy beyond that distance, and none from the galaxy's event on that slice meets ours: no future signal passes between them. The galaxy's past-inextendible worldlines end at the same initial singularity as ours, so it belongs to the same domain and to the same present [A2, Remark 10].

Remark 4 (Spacetime as representation). CP uses spacetime as special and general relativity do. Four-dimensional solutions, causal structure, light cones, and proper time describe the physics, and relativistic results are retained within the theory's stated regimes. CP denies one further interpretation, which neither the mathematics nor the geometry requires: that every event in a solution exists equally. The four-dimensional geometry represents how the present advances under the law [Remark 16]. Only the present exists, and the geometry on its slice belongs to it [Definition 6]. Curves, causal boundaries, and regions belong to the spacetime representation, and statements about them describe that representation.

Remark 5 (Univocity as the lever, with simultaneity downstream). CP places the question of presentism and relativity [51, 52, 57, 58] at the existence root. The operative commitment is univocity [A2], whose co-membership clause supplies the single actual present. The slice of the now is the downstream consequence supplied by A6, a neo-Lorentzian posit [15, 19].

2.3 The reality criterion

Something is real if and only if it is present, where the present is the one actual physical reality as it becomes [A1]. Definition 7 places it on the actual slice of the cosmic foliation. The criterion fixes what exists and, with it, where becoming occurs: the actual present is what becomes [55]. By the posit of the present truthmaker, present-tensed truths are made true by the actual present physical reality.

Definition 6 (Physical reality and the quantum state)**.** The theory distinguishes what exists from the nomic structure it instantiates.

- Σ, the *actual physical reality*. Within a causal domain, Σ is the universe as it exists now [A1]. It includes the geometry on the actual slice and its dynamical data, the actual values of matter and fields, and all physical records and traces. Σ is the concrete present and the sole concrete totality. The four-dimensional history represents its evolution and is not that totality.
- ω, the *quantum state*: nomic structure and a mathematical input to the present dynamics. It belongs to the present as that on which the present's change depends [A4], and it is not a second concrete object. It is nomic in that, with the adopted law of the actual physical variables, it fixes the probabilities with which those variables can change, and itself evolves by a law. Being nomic does not mean being unchanging. The alternatives it describes are not further actual worlds. In the unitary realization [Definition 1], the state of a subsystem is the conditional state that ω and the actual configuration of its environment fix; preparing a subsystem fixes that conditional state through the actual configuration, with no collapse, while ω evolves by its law.

Where the text writes ρ for a density operator in an operational identity, the identity holds for the quantum state and for an observer's epistemic description of it under ignorance of the actual physical variables. Σ is what exists.

Definition 7 (The present)**.** The present is the actual physical reality, which is primitive [A1]. The cosmic foliation places it on one slice, the actual slice [A6], and it intrinsically becomes as the cosmic parameter advances [A3]. Under its actuality description it is the real [Section 2.3].

Definition 8 (The past)**.** The past is not real: it is a present label for a configuration the present does not instantiate [A1]. A past event was real when it was the present. It cannot be changed, nothing stores it, and what is true of it now is fixed by the present and the law, under the posit of historical truth.

Definition 9 (The future)**.** The future is not real: what exists is the present [A1]. A course of change that the law fixes uniquely from the present is not an existing future.

The criterion sets the temporal asymmetry. The present cannot reach what it does not instantiate [Remark 51], the future does not exist, and the present alone is actual. Within a realization of the cosmic foliation, the criterion is total on the region covered by its successive spatial slices [Remark 19]. Space is the extent of the present. No container outlasts it.

Remark 10 (Accessibility is membership). A distant event is present if and only if it belongs to the one actual present [A2 and A3, Definition 7]. This membership is frame-independent and does not depend on whether a signal from the event has arrived. Accessibility defined by signal arrival would be limited to the observer's past light cone and would make becoming relative to the observer's location; CP defines it by membership in the actual present.

2.4 The global now

A dust grain in Andromeda and a walker on Earth are real on the same terms: each belongs to the one actual present. Univocity holds diachronically [A2]: as the present becomes, membership in it remains one fact, independent of vantage. Where domains overlap, their presents agree on the common events, so their foliations cut those events into the same slices, taken in the same order. An event that was present is not thereby co-present with what is present now.

Remark 11 (Explication of the global now). Axiom A1 posits one unfragmented present, and the reality criterion makes it the whole of what is real [Section 2.3]. The co-membership clause of A2 adds that membership in that present is the same from every vantage: all real events in the domain share one actual present.

Proof. Let P be the actual present of a causal domain [A1]. By the reality criterion an event of the domain is real exactly when it belongs to P, and by the co-membership clause of A2 membership in P is the same from every vantage. The real events of the domain are therefore one set, P, from every vantage. The argument uses nothing about which present is actual, so it holds as the present becomes, which is the diachronic reading. The argument uses no spacelike slice, no trichotomy, and no instant that transcends frames [Remark 12]. □

Remark 12 (Scope of Remark 11). Co-membership is membership under one truthmaker; co-location on a spacelike slice plays no part in it. Univocity delivers a present that is one and unfragmented. It does not deliver a spacelike slice. A6 supplies that geometric structure: on its admissible realizations every slice is a spacelike Cauchy surface. A Cauchy surface is a slice of spacetime that every inextendible causal curve meets at exactly one event [39]. Remark 11 uses A1 and A2 and the reality criterion. The incompatibility of Remark 13 uses A1 to A3 and the theorems of Malament and Stein. Neither uses the premise of the cosmic foliation or the choice of realization law, and the resolution of the incompatibility rests on A6. Within a realization, both are claimed on the region its slices cover.

2.5 The incompatibility

Remark 13 (The incompatibility, and where CP differs). CP requires one actual present and a single, global becoming [A1 to A3]. Which distant events belong to the same present must therefore have an objective answer, independent of the observer.

Special relativity does not supply that answer from causal structure alone. In Minkowski spacetime, Malament shows that, under his stated conditions, any equivalence relation defined from causal connectibility that relates two distinct events must relate every event to every other event [43, Prop. 3]. Such a relation cannot identify one present. Stein likewise shows that becoming defined from causal structure alone is relative to a point and never global [57, 58]. A single, global becoming therefore cannot be defined from causal structure alone.

CP accepts these mathematical results. It rejects the assumption that causal structure must be the only structure available to define the present. A2 requires one global, univocal becoming, and A6 supplies the additional structure it needs: the cosmic foliation.

Observers in different frames can still disagree about the order in coordinate time of spacelike-separated events. They do not disagree about which events actually share the present. Membership in the actual present, and the order of its becoming, are fixed by the cosmic foliation, independently of any observer's frame.

In curved spacetimes with special symmetry, such as an exact Friedmann-Lemaître-Robertson-Walker (FLRW) universe, a natural cosmological slicing is already distinguished. CP nevertheless adopts its cosmic foliation as part of the theory and takes none of its ontological significance from that symmetry. The incompatibility established here concerns Minkowski spacetime.

2.6 Individuation by causal domain

The present is individuated by causal domain [A2], with one actual present per domain of common origin, a present that does not branch [A1].

3 Intrinsic Becoming

The present does not stand still: it becomes. Becoming is the theory's one addition to the present, and it is a primitive posit. It is the advance of actuality itself: the whole present advances at once, in the order the cosmic foliation fixes, for every observer together [Remark 18]. It is distinct from the dynamics, which changes what the present contains. A particle's position can wander while its quantum state stands still, and that wandering is dynamics. It is not becoming [Section 4].

Axiom A3 (Becoming)**.** *The actual present intrinsically becomes, and the advance of the present is the advance of actuality. Becoming is not passage through an existing time dimension, nor replacement of one coexisting slice by another. No fundamental duration is assigned to the advance.*

What has become was actual and cannot be undone [Remark 51]. The universe is one actual present that becomes, and the cosmic foliation represents the order of its advance [A6]. Becoming acts on no quantum state. Time dilation follows the usual laws of relativity [Remark 18].

Remark 14 (What the ordering delivers). (a) In the representation, every inextendible worldline the foliation covers meets each of the same ordered slices exactly once, since every slice is a Cauchy surface [A6, Remark 12], so the order of the present's advance is common to them. (b) The present advances continuously. A complete instantaneous configuration with its dynamical data is a mathematical description of the present, evaluable on any slice, and it describes continuing change without suspending it. CP posits no discrete global update. Discrete physical outcomes occur where the physics has them.

Axiom A4 (Complete present)**.** *The present contains everything its change depends on, including matter, fields, gravitational geometry with its dynamical data, and the quantum state. No earlier configuration is an input to how it changes.*

In statements of the law, a complete present is a specification of everything on which the present's change depends. It is a kind of present, never a further existing present.

Definition 15 (Continuation)**.** A *continuation* of a complete present is a course of change that the law admits from it: in the representation, a sequence of complete presents on later slices, beginning with it, each of whose steps the law admits. Where the law's transition probabilities are atomic, the law admits a step when it assigns it positive probability, as for the histories of the present fiber [Definition 39]. A continuation is *determined* when the law admits no other. Continuations are possibilities the law admits [Definition 9].

Present sufficiency follows. For a fixed realization [Definition 1], the change of the present is a function of the complete present alone. Two identical complete presents under one law have the same distribution of continuations [A5], a single continuation where the law is deterministic. In the classical Einstein-matter regime, the well-posed Cauchy problem on the slice governs this change, with the geometry and its extrinsic curvature as the gravitational data [17] [Section 4.1]. The quantum state likewise carries dynamical data: for a free particle, Re^{ikx} and Re^{-ikx}, with R real, have the same position density $|\psi|^2$ but opposite probability currents. Present sufficiency does not entail historical determinacy; whether antecedents are fixed depends on the complete present and the law [Lemma 41].

Remark 16 (No temporal surrogates). CP introduces no fundamental variable whose sole role is to encode past truth. Ordinary records, memories, and physical traces are unaffected. Every present variable is independently warranted as a physical, dispositional, or nomological constituent of the complete present [A4]. An object fails the policy if its only role is to stand in for the past or the future. A four-dimensional solution of the field equations is admitted as a representation, without the reading on which it catalogues equally existing times.

Remark 17 (The advance of actuality). Irreversibility belongs to actuality, which advances as one global present [A2 and A3]. The earlier present was actual when it was present; it no longer exists. The quantum state need not evolve irreversibly: in the unitary realization its evolution is invertible. The advance is not an accumulation, and no quantity of actuality grows. Becoming continues even when entropy remains constant and, under a law with unique predecessors, when no past truth is lost [Lemma 41 (iii)]. Settling and decay need no asymmetry in the law: under a law in detailed balance with its stationary distribution, a proposition about the present at one parameter can have no truth value before that present, be true at it, and have none again afterwards [Lemmas 41 and 42]. Becoming, not the law, orients the difference.

3.1 Orientation

Becoming carries its own orientation. The present advances in the order the foliation fixes [A3 and A6], and no thermodynamic input fixes that order. No measurement, dephasing, or decoherence process initiates or orients becoming, and becoming needs no observation, no monitor, and no localization length. A stationary vacuum also becomes. The thermodynamic arrow is statistically aligned with the advance, on the premise of Remark 21.

4 Physical Dynamics and the Cosmic Foliation

The present becomes, and what it contains changes: matter moves, fields and the quantum state evolve, and the geometry changes with them. Becoming is the advance of actuality [A3]. The change of the contents is the dynamics, and one law governs all of it. Where the present lies in the geometry is a separate question, and the theory answers it with its one geometric posit beyond general relativity: a cosmic foliation whose actual slice represents the present, posited for global becoming, co-presence, and the order of the present [A6].

Axiom A5 (One dynamics)**.** *The complete present changes according to one mutually consistent physical dynamics, which governs matter, quantum fields, gravity, and causal structure together. The law of the actual physical variables is stochastic: for some physically realizable complete presents it fixes a distribution of continuations and not a single one.*

The temporal ontology and A5 select neither collapse nor its absence [Definition 1]. Stochastic collapse of the state with a mass-density primitive ontology [3] is a candidate realization: its collapsed state is Markov, and its actual variables are the mass density on each slice. A flash ontology, which supports a relativistic collapse theory without a preferred foliation [65], does not meet A7: a slice almost surely contains no flash, so no readout is encoded in the actual variables of the present [3]. Presentism thus selects primitive ontologies carried on the slice.

The stochastic clause is an adopted law of the actual physical variables; A1 to A4 do not entail it. Bell-type jump processes realize it on finite configuration spaces and, through creation and annihilation, in regularized field theory [Section 5.2]. The placement law of the foliation [Remark 19] and the classical Cauchy problem [Section 4.1] are deterministic given their data. Where the coupled dynamics lets a random change of the actual matter variables change those data, the slices and the geometry inherit that chance; in the $\mathbb{T}^3$ minisuperspace sector the geometry does [Proposition 38].

In a Bell-type realization the quantum state follows its unitary law and fixes the transition rates of the actual configuration, and the rates govern the possible successors without selecting one [Section 5.2]. Neither a universal noise term nor an additive correction to classical equations follows from A5, and CP posits no fluctuation beyond the quantum dynamics of its realization.

Example. The stochastic clause does not require jumps. Take a particle of mass m in a harmonic potential of frequency Ω, in its ground state, with position density $\rho(x) = |\psi(x)|^2 = e^{-x^2/\ell^2}/(\sqrt{\pi}\,\ell)$ and $\ell^2 = \hbar/(m\Omega)$; the state is stationary and carries no current. Let the actual position follow the Nelson-type diffusion $dX = -\Omega X\,dt + \sqrt{\hbar/m}\,dW$ [Section 5.2]. Its probability flux $-\Omega x\rho - \frac{\hbar}{2m}\rho'$ vanishes identically, so the law carries the density $|\psi|^2$ to itself. The quantum state does not change, the present fixes a distribution of continuations of the position, and the position changes. This is the dynamics of A5. It is not the becoming of A3.

Axiom A6 (Cosmic foliation (neo-Lorentzian posit))**.** *The existence of a global slice of the now is part of the posit. The domain carries a primitive temporal structure of neo-Lorentzian type [19]. Within the cosmic foliation, the one actual present of Remark 11 is represented on a spatial slice S_τ of the realized geometry. Its placement obeys a causal law [Remark 19]. The slice is not the concrete present itself, but its place in the spacetime representation.*

The cosmic parameter τ is a continuous label carried by the present. It is neither a discrete tick nor a measure of universal elapsed time. It orders the slices but does not measure proper time along an arbitrary worldline. It strictly increases as the present becomes. Earlier slices represent presents that were actual; they do not exist alongside the present now.

In the four-dimensional representation, these slices form a family $\mathcal{F} = \{S_\tau\}$, describing their ordered placement. This family is mathematical and contains no simultaneously existing presents. The foliation specifies the collection and ordering of the slices, and the present's value of τ identifies which slice represents it; the foliation alone does not identify the actual member. It implies no external clock. Clocks measure physical change, and coordinates label it. The metric of the realized geometry determines local causal structure. A realized geometry is an admissible realization of this posit only if every slice of $\mathcal{F}$ is a spacelike Cauchy surface of the domain, which requires the domain to be globally hyperbolic [39].

Global hyperbolicity is exactly the condition for a foliation by spacelike Cauchy surfaces to exist: every globally hyperbolic spacetime admits a smooth time function whose levels are spacelike Cauchy surfaces [10], and it has infinitely many such time functions. In Minkowski spacetime the time coordinate of every inertial frame is one, and each frame slices spacetime differently. Only one foliation carries the present; the level sets of any other foliation label the realized geometry and are not presents. The neo-Lorentzian literature calls such structure a preferred foliation [19], but in the theory there is no other foliation for it to be preferred over. Of its slices, the earlier represent presents that were actual, the later represent nothing that exists, and one, the actual one, represents the present. Everything that happens happens in the present.

Remark 18 (No worldline leaves the present). Every inextendible causal curve in the region the slices sweep meets each slice at exactly one event [A6]. However fast a body moves, its worldline meets every slice. Motion and gravity change how much proper time a clock records, never which slice represents the present. In the representation the worldlines of twins who part and reunite meet the same slices, and the travelling twin's clock records less proper time between departure and reunion [A3]. The travelling twin has counted less time and has not left the present. At the reunion both twins are on the actual slice, and neither meets a past or future version of the other. No future-directed causal curve returns to an earlier slice either, since the cosmic parameter strictly increases and closed causal curves are excluded [Remark 21]. Nothing done within the present moves any part of it to another slice: every interaction acts on the contents of the actual slice, and only becoming advances the present, for every observer at once.

Relativity fixes how much each clock records, with no observer's frame preferred for that; the cosmic foliation places every reading, in any frame, on the slice of the one present [A1], and that present is what the theory means by now.

Remark 19 (The foliation is posited structure). The cosmic foliation is primitive: it is not a further fundamental field, it carries no stress-energy, and the field equations do not determine it. Under its causal placement law, the placement at each later event depends only on the data of the present slice that lie in that event's causal past. The lapse driver below meets this law on homogeneous FLRW and is adopted as its realization on perturbed FLRW; for the coupled dynamics the law is a requirement on the realization [Section 7.1]. The slices sweep the region $M_{\mathcal{F}} := \bigcup_\tau S_\tau$ of the realized geometry, a four-dimensional representation [Remark 16]. Singular boundaries of that geometry are not presents.

On small perturbations of spatially flat FLRW spacetime on $\mathbb{T}^3$ with $\Lambda > 0$ and dust or a perfect fluid of sound speed $0 < c_s^2 < 1/3$, a background that is nonlinearly future stable [37, 53, 56], the adopted realization of the causal placement law is a causal lapse driver,

$$(\partial_\tau - \mathcal{L}_\beta)N = -N^2(K - K_\star(\tau)),$$

with N the lapse and β the shift of the slices, $K = \operatorname{tr} k$ their mean curvature for the extrinsic curvature $k_{ij} = -\nabla_i n_j$ of the future unit normal n (an expanding slice has $K < 0$), and $K_\star$ a prescribed function of the slice parameter alone. Up to the lower-order term in $K_\star$, the driver is the Bona-Massó slicing condition with $f = 1$ (harmonic slicing), hyperbolic with gauge speed $N\sqrt{f} = N$, the speed of light. An elliptic condition, such as maximal slicing with infinite gauge speed [13], fails the causal placement law. A realization whose ordering is supplied by extra field content, such as a clock field coupled into the field equations, is inadmissible [A5]. The FLRW proper-time slicing is the fixed point $K = K_\star$: with zero shift and $K_\star(\tau)$ the homogeneous mean curvature, the right-hand side vanishes, and $N = 1$ gives proper time. Hyperbolicity does not single out a foliation: $K_\star$ and the driver's Cauchy data (the initial slice and lapse) are realization inputs and add no further primitives [Section 7.1]. The lapse belongs to the complete present [A4], and the driver reads $K_\star$ at the cosmic parameter of the present slice, a value posited with the slice [A6] and so a feature of the present [Definition 7]. Only affine relabelings $\tau \mapsto a\tau + b$ keep the driver's form, and their scale is fixed with $K_\star$.

Proposition 20 (The homogeneous driver). *Let H be the expansion rate of a spatially flat FLRW geometry as a function of proper time on $(0, \infty)$: differentiable and nonincreasing, with nondecreasing derivative. On its homogeneous slices with zero shift, with $t(\tau)$ the proper time of the slice at parameter τ and $K_\star(\tau) = -3H(\tau)$, the driver reads*

$$t' = N, \qquad N' = 3N^2(H(t) - H(\tau)).$$

For all $\tau_0, t_0, N_0 > 0$ it has exactly one solution on $[\tau_0, \infty)$ with $t(\tau_0) = t_0$ and $N(\tau_0) = N_0$. Along it the lapse lies between two positive bounds fixed by (τ_0, t_0, N_0), and t increases strictly through every value in $[t_0, \infty)$.

The proof is in Appendix A.9. The hypotheses hold on every flat background whose source is a set of noninteracting perfect fluids of positive mass densities ρ_i and constant equations of state $w_i \geq -1$, radiation, dust, and Λ included, so in every era of such a cosmology: $H' = -4\pi G \sum_i (1+w_i)\rho_i \leq 0$ and $H'' = 12\pi G\, H \sum_i (1+w_i)^2 \rho_i \geq 0$. The perturbed developments are treated in Section 7.4.

In the classical Einstein-matter regime [Section 6] the foliation enters no Einstein equation, so the metric is that of general relativity, its parametrized post-Newtonian preferred-frame parameters vanish ($\alpha_1 = \alpha_2 = \alpha_3 = 0$), and every preferred-frame bound is met identically [67]. A joint realization lies outside this regime when its geometric law depends on the lapse [Section 7].

Remark 21 (The Gödel reading). Global becoming is posited, and its ordering is the posited cosmic parameter, a global time function on every admissible realization. Global hyperbolicity, required by A6, excludes closed causal curves there, so the slices cannot sweep a spacetime whose events carry closed causal curves, as Gödel's rotating cosmology [35] does. The theory describes the actual universe: it entails that no closed causal curve passes through any present event, and observing one refutes the theory. The realized geometry is the globally hyperbolic region $M_{\mathcal{F}}$ the cosmic foliation sweeps, within the domain its common origin fixes [Definition 2, A6]; an extension beyond a Cauchy horizon belongs to it only if it admits that foliation. No slice recurs, since the cosmic parameter strictly increases. A prior slice actual beside the present one would be a second present [A1]; a present record of an earlier event is neither a recurrence nor a second present. The present advances in the order of the slices. The thermodynamic arrow is statistically aligned with that advance under the usual assumptions of statistical mechanics and the one premise CP adopts, the low-entropy premise: in the representation, the law of the present nomic state traces the domain back to an initial condition of low coarse-grained entropy of matter and geometry [2, 50]. Entropy does not define the cosmic parameter.

4.1 Microcausality and the cosmic foliation

CP poses evolution as a Cauchy problem on the cosmic foliation (ADM 3+1 with the cosmic slicing physical) on every admissible realization of A6 [17]. The field theory keeps its microcausal dynamics: spacelike-separated field operators (anti)commute, and local interactions respect microcausality. Entanglement in the quantum state, and correlations among the actual variables, can extend across spacelike separation. The foliation adds no Lorentz-violating field content and replaces no covariant matter dynamics. Where the law of the quantum state is that of $\mathbb{V}$, the spacelike order of the present enters no accessible marginal of the state [Remarks 27 and 28].

In exact FLRW with comoving radiation and the fixed-point inputs (homogeneous $K_\star$ and initial slice, $N = 1$, zero shift), the comoving slicing solves the driver, and the slices of the cosmic foliation coincide with the simultaneity slices of the radiation rest frame [Remark 19]. The fixed point by itself identifies neither the radiation rest frame nor a unique foliation [Remark 19].

5 Measurement within Becoming

A detector clicks and a pointer settles. Nothing in this makes anything actual. The present is already actual and already becoming [A1 and A3], and a measurement is a physical interaction within it. One present physical reality is actual [Definition 6], and actuality belongs to it and its becoming, never to the structure of the quantum state. Definite values belong to the realization's actual physical variables rather than to every quantum observable, so their definiteness requires no collapse. In the unitary realization the quantum state does not collapse [A5]. Three distinctions fix what this asserts.

(i) A component of the state that does not describe the actual outcome is still part of the state. It stays in the state's evolution and can recohere, as in the quantum eraser [54].

(ii) Definite actuality is axiomatic [A1]. The statistics of the Born rule and accessible correlations are owed by the physical realization [A7]. Presentism alone does not supply them.

(iii) Continuity of becoming leaves physical transitions in place. A fast detector transition with a discrete reading is ordinary physics inside the one dynamics.

Spontaneous-localization theories, Ghirardi-Rimini-Weber (GRW) and Continuous Spontaneous Localization (CSL) among them, modify quantum dynamics and can produce observable

localization, heating, or radiation [34]. A collapse-type deviation from standard quantum mechanics would refute every unitary realization. The temporal ontology does not rest on that dynamics [A5].

5.1 Record formation

A record forms when a register couples to a further degree of freedom and each pointer value leaves its own trace there. In the unitary realization the coupling is unitary. The configuration basis is posited with the realization, and environmental einselection explains the stability of records in it [68, 69].

Definition 22 (Record formation)**.** A *register* is a physical degree of freedom with Hilbert space $\mathcal{H}_S$ (a detector, an environmental degree of freedom, or a memory bit), carrying a pointer family $\{P_k\}_{k=1}^{d}$ of mutually orthogonal projections with $\sum_k P_k = I_S$. *Record formation* is the unitary coupling of the register with a record factor $\mathcal{H}_E$ in a ready state E_0 that carries $|\psi\rangle_S|E_0\rangle$ to $\sum_k P_k|\psi\rangle_S|E_k\rangle$ for every $|\psi\rangle_S \in \mathcal{H}_S$; it induces on $\mathcal{H}_S$ the isometry $V|\psi\rangle_S \coloneqq \sum_k P_k|\psi\rangle_S \otimes |E_k\rangle$ and the decoherence channel $\rho_S \mapsto \mathrm{tr}_E(V\rho_S V^\dagger)$. The record states E_k are unit vectors. The exactly dephased channel is $\mathcal{D}(\rho_S) = \sum_k P_k\rho_S P_k$. Distinct registers are tensor factors of their own, or local algebras of distinct regions.

Definition 23 (Born weight)**.** The *Born weight* of a projection P, or of an outcome operator E, in the quantum state ω is $\omega(P)$ or $\omega(E)$; for a vector state ψ it is $\langle\psi|P|\psi\rangle$. On a finite configuration space the Born weight of a configuration i is $|\psi_i|^2$, and the Born weights of all configurations form the *Born distribution* on the slice. On a continuous configuration space the Born distribution has the *Born density* $|\psi(x)|^2$. The Born weight of a complete present is that of its configuration. A Born weight is a number fixed by the quantum state, not a value of the actual physical variables. A *Born node* is a parameter at which a Born weight vanishes while vanishing on no open interval around it.

Decoherence yields the sector family $\{P_k|\psi\rangle_S|E_k\rangle\}$ with Born weights $\langle\psi|P_k|\psi\rangle$. For orthogonal records the induced channel is the exact dephasing channel $\mathcal{D}$, the deterministic ensemble channel the state undergoes [49, 69]. The unitary coupling realizes it as its Stinespring dilation [59]. Markovian dephasing, with generators in Gorini-Kossakowski-Sudarshan-Lindblad form [36, 42], reaches $\mathcal{D}$ only asymptotically. On a finite-dimensional register a semigroup with bounded generator is invertible at every finite time, and exact dephasing is not. Overlapping records multiply the coherence between sectors j and k by the overlap $\langle E_k \mid E_j\rangle$, reducing it without removing it.

5.2 Born statistics

A readout is a value of the actual physical variables, and one axiom ties it to the Born weights.

Axiom A7 (Quantum consistency)**.** *The present quantum state fixes the probabilities and joint correlations of the readouts of operationally accessible measurements through the Born rule of the realization's instrument. A measurement is operationally accessible when its outcome is encoded in distinguishable values of the actual physical variables, which are its readout.*

Agreement with measurement is the test of a realization: a realization whose Born probabilities or joint correlations disagree with those measured is refuted.

Definition 24 (Quantum equilibrium)**.** The actual physical variables are in *quantum equilibrium* on a slice when their distribution there is the Born distribution of the quantum state on that slice. This distribution is a measure of typicality over the values a present can have, not a frequency in one history. A realization is in quantum equilibrium when this holds on every slice.

Its law *preserves* quantum equilibrium, or is *equivariant*, when it carries quantum equilibrium on one slice to quantum equilibrium on every later one. Every other distribution is *quantum nonequilibrium*.

The readout clause puts the content of A7 on the actual variables. The realization must make the distribution of the realized readouts match the Born probabilities its instrument assigns. An instrument has outcome operators $E_z = \sum_\alpha L_{z,\alpha}^\dagger L_{z,\alpha}$ with $\sum_z E_z = I$. Its Born rule assigns readout z the probability $\omega(E_z)$, and its state-update maps fix the joint correlations of successive readouts. In quantum equilibrium the unitary realization reproduces the observable marginal probabilities of unitary quantum mechanics [Remark 27]. A collapse realization carries the departures its collapse law predicts, which experiment bounds [7].

In the developed case of the unitary realization the actual configuration follows a Bell-type jump process [9]. It jumps at rates the present quantum state fixes. A jump is a transition the complete present does not fix, and it changes the configuration, never the quantum state, so it is no collapse. Equivariant rates carry a Born-distributed configuration to a Born-distributed configuration [Lemma 26]. On finite configuration spaces, over parameter intervals where the rates are integrable, this proves Born equivariance and shows that the stochastic clause of A5 can be met. Dürr, Goldstein, Tumulka, and Zanghì [29] give methods for constructing Born-distributed Markov processes in a broad class of regularized quantum field theories. They demonstrate the construction explicitly for a second-quantized Dirac field in an external electromagnetic field. The unitary realization adopts this class of processes without reconstructing it. Where such a process exists, the Born distribution is, in quantum equilibrium, the one-time marginal of its measure over histories. There it is also the distribution of operational readouts recorded in the actual configuration variables [Section 7.2].

Bohm-type guidance [11, 12] is equivariant [25] and deterministic. With Born-distributed initial data it meets A7 in its nonrelativistic domain. It does not meet the stochastic clause. At fixed particle number the minimal Bell rates approach it in the continuum limit [29]. There the clause is met by symmetric rates above the minimal ones, scaled so that they survive that limit; these rates give Nelson-type diffusion [29, 48]. In field theory the clause is met by the jumps that create and annihilate particles. Relativistic Bohm-type models need a preferred foliation. One proposal fixes it covariantly from the wave function [30], and the lapse driver of Remark 19 fixes it by a causal evolution law from an initial slice. Let the background be fixed and the lapse of the driver stay between positive bounds [Section 7.4]. The cosmic foliation then meets the conditions the hypersurface Bohm-Dirac models place on theirs. These are smooth spacelike Cauchy slices of a time function, on a curved background, placed by a law that does not involve the actual configuration [28, 64]. There Born equivariance is established for noninteracting Dirac particles in an external field [28].

Definition 25 (Jump law)**.** On a finite configuration space let the quantum state evolve by $\dot\psi = -iH\psi$, in units with $\hbar = 1$, with H Hermitian, and let $J_{ij} = 2\,\mathrm{Im}(\bar\psi_i H_{ij}\psi_j)$ be the probability current from configuration j to configuration i. The *jump law* moves the configuration from j to i at rate $\max(0, J_{ij})/|\psi_j|^2$, and at rate zero where $\psi_j = 0$. The *law with the further rate* adds the rate $\lambda_{ac}|\psi_a||\psi_c|/|\psi_a|^2$ from a to c, zero where $\psi_a = 0$, for a constant matrix λ that is symmetric, nonnegative, and positive exactly on the pairs $a \neq c$ with H_{ac} not identically zero.

Lemma 26 (Equivariance of the jump law)**.** *On a finite configuration space let the configuration follow the jump law [Definition 25]. At the Born weights $P_i = |\psi_i|^2$ the master equation gives $\dot P_i = \sum_j J_{ij}$, which is $\frac{d}{dt}|\psi_i|^2$, for every configuration i, and on any time interval on which the rates are integrable the Born weights are the unique solution with Born initial data. On two configurations with $H = \sigma_x$ and $\psi(0) = (1, -i)/\sqrt{2}$, for $0 \le t < \pi/4$ the rate from configuration 0 to 1 is $2\cos 2t/(1 - \sin 2t)$, the reverse rate is zero, and from configuration 0 the probability of a jump by time t is $\sin 2t$, strictly between 0 and 1 for $0 < t < \pi/4$.*

The proof is in Appendix A.2.

Example. The witness of Lemma 26 shows a future-tensed proposition without truth value and a lost past truth value in one law. Take the slice parameters 0 and $\pi/12$, reading the lemma's time as the slice parameter, with the data given on the slice at 0. Take the complete present at each parameter to be the configuration together with the state of the lemma, which evolves by its own law whatever the configuration. Each configuration has Born weight $\frac{1}{2}$ at 0. From configuration 0 the law reaches configuration 1 by $\pi/12$ with probability $\sin(\pi/6) = \frac{1}{2}$ and stays with probability $\frac{1}{2}$. So at a present at 0 with configuration 0 the proposition that the configuration at $\pi/12$ is 1 has no truth value [Section 8]. It holds on some admitted continuations and fails on others. From configuration 1 the reverse rate vanishes and the law stays with probability one; the Born weights at $\pi/12$ are $\frac{1}{4}$ and $\frac{3}{4}$. At a present at $\pi/12$ with configuration 0 the one admitted history starts in configuration 0. The proposition that the configuration at parameter 0 was 0 is then true. At a present at $\pi/12$ with configuration 1, histories starting in both configurations, each of positive weight, are admitted. That proposition then has no truth value [Lemma 41 (iv)]. In neither case is it false. Where it has no truth value it is indeterminate. It is not partly true, because a conditional Born weight is not a degree of truth.

Remark 27 (Marginal coincidence, a consistency observation). Let a realization evolve the quantum state by the law of unitary quantum mechanics $\mathbb{V}$, $\omega_t = \omega_t^{\mathbb{V}}$ for every initial state ω and every time t.

(i) Every functional of the state takes at ω_t its value at $\omega_t^{\mathbb{V}}$. This holds in particular for every accessible marginal $\omega_t|_{\mathcal{A}}$ and for the mean energy $\omega_t(H)$, whenever it is defined and finite.

(ii) If the realization is in quantum equilibrium [Definition 24], so that the distribution P_t of its designated actual variables on an outcome space is the Born statistics $\mathrm{Born}(\omega_t)$ at every t (an equivariant law with Born-distributed initial data [25]), then $P_t = \mathrm{Born}(\omega_t^{\mathbb{V}})$ at every t, and the same holds for every coarse-graining of the outcome space.

Proof. Part (i) substitutes $\omega_t^{\mathbb{V}}$ for ω_t. For part (ii), part (i) applied to $\omega \mapsto \mathrm{Born}(\omega)$ gives $\mathrm{Born}(\omega_t) = \mathrm{Born}(\omega_t^{\mathbb{V}})$, and a coarse-graining pushes equal distributions forward to equal distributions. □

Remark 28 (What carries each part). The unitary realization supplies the hypothesis of the observation wherever its quantum state obeys the law of $\mathbb{V}$ independently of the actual physical variables [A5]. This holds at a fixed background. It also holds in the weak-field regime where the adopted dynamics is the unitary linearized quantum theory, with the metric perturbation carried by the quantum state [Section 6]. Where the geometry is a variable of the quantum state, the state obeys constraint equations and the remark does not apply. The same identity holds for the constraint theory in place of $\mathbb{V}$, but only where its marginals are defined, and its density is not normalizable [27]. A state has a law that depends on the configuration when it evolves on an actual geometry responding to actual matter. For the coupled dynamics the identity of laws is unproved [Section 7.5].

Part (ii) adds quantum equilibrium, a condition on the unitary realization's initial distribution that does not follow from A5. With readouts recorded in the actual configuration it is a sufficient condition for A7.

Lemma 29 (Born frequencies in persisting records)**.** *On a finite configuration space let $\rho_0, \dots, \rho_N$ be the Born weights at chosen parameters $t_0 < \cdots < t_N$, let transition probabilities T_j from t_j to t_{j+1} carry ρ_j to ρ_{j+1}, and let a history $(x_0, \dots, x_N)$ have weight $\rho_0(x_0)\prod_{j<N} T_j(x_j, x_{j+1})$. A record written at t_i is a function of the configuration that every transition of positive probability after t_i preserves.*

(i) For records $R_1, \dots, R_m$, each read at its write parameter, the weight of the histories with record values y is the Born weight at t_N of the configurations with record values y. If that

weight is $\prod_i w(y_i)$ for a probability distribution w, then for every set A of outcomes and every $\varepsilon \geq 0$ the histories whose frequency of outcomes in A differs from $\sum_{a\in A} w(a)$ by at least ε have weight at most $2e^{-2m\varepsilon^2}$.

(ii) Let earlier records with values y and a new record with value o persist to t_N, let the Born weights at t_N be those of $VU\psi$, with ψ the state before the readout, U the readout, and V the later evolution, both norm preserving, let U commute with the projection Π_y onto the configurations with earlier record values y, and let V commute with Π_y and with the projection Π'_o onto those with new record o. Then the histories with earlier records y and new record o have weight $\|\Pi'_o U \Pi_y \psi\|^2$, and those with earlier records y have weight $\|\Pi_y \psi\|^2$.

(iii) For records $R_1, \ldots, R_m$, each read at its write parameter, let p_k be the weight of the histories with $R_k \in A$ among those with the same earlier records $R_1, \ldots, R_{k-1}$. For every $\varepsilon \geq 0$ the histories whose frequency of outcomes in A differs from $\frac{1}{m}\sum_k p_k$ by at least ε have weight at most $2e^{-2m\varepsilon^2}$. If the kth readout U_k commutes with the projection Π_y onto every value y of the earlier records and its later evolution satisfies the hypotheses of (ii), then for each earlier record string y of positive weight $p_k = \|\Pi'_A U_k \Pi_y \psi_k\|^2 / \|U_k \Pi_y \psi_k\|^2$.

Under the jump law [Definition 25], a function R of the configuration with $H_{ij} = 0$ whenever $R(i) \neq R(j)$, at every parameter after t_i, is a record written at t_i on every history of positive weight. No jump of positive rate changes its value, and the Born weight of each of its values is constant. Consider a Hamiltonian that agrees on each of finitely many intervals with a Hermitian Hamiltonian continuous in the parameter. On each interval let each Born weight be positive throughout or zero throughout. Its later intervals can depend on the records already written and couple no values of them. For such a Hamiltonian the transition probabilities of the jump law itself, through the interval ends and the Born nodes [Definition 23] that end each write, meet the hypotheses of (i) to (iii). The record strings of a sequence of measurements, adaptive choices included, then carry the Born weights of their record-projected states. The proof is in Appendix A.6.

5.3 Operational no-signalling

At a fixed background, setting choices do not signal. Two steps carry this, one on the quantum state and one, which needs quantum equilibrium, on the actual variables that carry the readouts. A third finds where the Bell correlations reside.

First, the state. Let O_A and O_B be spacelike-separated regions of a net of local algebras with $\mathcal{A}(O_A) \subseteq \mathcal{A}(O_B)'$, let ω be a state, and let Alice's setting θ index any finite normalized local family $\{M_k^\theta\} \subset \mathcal{A}(O_A)$, $\sum_k (M_k^\theta)^\dagger M_k^\theta = I$. Then $\sum_k \omega((M_k^\theta)^\dagger E M_k^\theta) = \omega(E)$ for every $E \in \mathcal{A}(O_B)$, with no dependence on θ [Lemma 54, Proposition 55, and Corollaries 56 and 57]. The statement is carried in normal functionals and uses no tensor factorization.

Second, the readouts. Let the unitary realization be in quantum equilibrium, with the spectral projections of its designated actual variables at Bob's region in $\mathcal{A}(O_B)$. Let Alice's setting be produced by a generator in a product state with Bob's region, in quantum state and actual configuration alike. Let the setting be kept as a record in the generator's configuration through Bob's readout. By part (ii) of Remark 27, the realization gives those variables, and every readout coarse-grained from them, the Born distribution of Bob's marginal. The first step leaves that marginal unchanged, so a setting does not signal through Bob's readouts either.

Quantum equilibrium cannot be dropped from this step. Take two qubits in the state $(|00\rangle + |11\rangle)/\sqrt{2}$, the actual configuration distributed as $(p, 0, 0, 1-p)$ over $00, 01, 10, 11$, and the jump law [Definition 25]. Alice either leaves her qubit alone or applies σ_x to it for time $\pi/4$. Bob then applies σ_x to his for time s, $0 < s < \pi/4$. Each operation moves the configuration only between two configurations whose current keeps one sign. The depleting configuration therefore stays with probability equal to the ratio of its Born weights. The filling one stays with certainty,

as in the proof of Lemma 26. Bob reads 0 with probability $p + (1-2p)\sin^2 s$ in the first case and $p + \frac{1}{2}(1-2p)\sin 2s$ in the second. The two differ by $(1-2p)\sin s\,(\sin s - \cos s)$, which vanishes exactly at the equilibrium value $p = \frac{1}{2}$. In this model, as in pilot-wave theory [66], a nonequilibrium distribution carries Alice's setting to Bob's readouts and fails A7 at them. So the model with that distribution is no admissible realization of the theory. Observed quantum nonequilibrium in the actual variables that carry readouts would violate A7, an axiom, and so refute the theory, not one realization of it.

Third, the Bell correlations. Quantum equilibrium enters again, now as a hypothesis on the data both wings share.

Lemma 30 (Equilibrium independence)**.** *Partition all systems into the system carrying λ (all the shared data on which the outcome laws of both wings depend apart from the settings) and the remaining systems, the setting generators among them, with the actual variables taken as coordinates in a product basis across the partition. Let the quantum state on a slice be a product across the partition, let the actual variables be distributed by quantum equilibrium, let the Hamiltonian be a sum of terms acting on one side each until the settings (a, b) are produced, and let the settings be produced from the generator side alone. Let the configuration local to each wing, its setting generator and setting included, be absorbed into that wing's response, and let the two local configurations be jointly independent of each other and of λ.*

(i) The distribution of λ does not depend on the settings: $\rho(\lambda \mid a, b) = \rho(\lambda)$.

(ii) For a model whose outcome law factorizes given λ, with outcomes in $[-1, 1]$ and $\delta_{ab} = \frac{1}{2}\sum_\lambda |\rho(\lambda \mid a, b) - \rho(\lambda)|$, the CHSH combination obeys $|S| \le 2 + 2\sum_{a,b}\delta_{ab}$, so $|S| \ge 2\sqrt{2}$ requires $\delta_{ab} \ge (2\sqrt{2} - 2)/8$ for some setting pair.

The proof is in Appendix A.7.

The hypotheses of Lemma 30 are a sufficient condition, which neither A4 nor A7 guarantees. Under them, the measured Bell-CHSH violation requires that the outcome law fail to factorize when conditioned on λ and the local configurations. That failure is ontological nonlocality among the actual physical variables. The complete present admits no future input that could carry the correlation in its place [A4], and the realization must stay operationally no-signalling [A7, Section 7.1].

The first step leaves room for this nonlocality, since the no-signalling identity constrains only the Born-weighted average over Alice's outcomes. Remote values conditioned on an outcome can depend on her setting and her outcome [Remark 58]. Bell-type jump rates depend on the joint configuration through the present state [Lemma 26], as Bohm-type guidance does [25]. The two-qubit model above shows this in quantum equilibrium. At $p = \frac{1}{2}$ Bob's readout probability is the same under both of Alice's settings. Yet the probability that his bit at s differs from its value when his operation begins is $\sin^2 s$ in the first case and $\frac{1}{2}\sin 2s$ in the second.

Three weakenings of the lemma's hypotheses fail. The product hypothesis concerns the quantum state of the shared data relevant to the outcomes, apparatus correlations included. Factorized Born weights do not suffice, because the factorized weights of an entangled state need not survive a local rotation. Separability does not suffice either: in $(|000\rangle + |111\rangle)/\sqrt{2}$ the source and the generator values are perfectly correlated although their reduced state is separable. The local configurations must be jointly independent, and independence variable by variable falls short. A generator variable of one wing equal to the parity of both settings, independent of each setting alone, reaches $S = 4$ with local responses. Relaxed Bell inequalities quantify the measurement dependence a given violation requires, in distance and information measures of their own [38]. The bound in part (ii) is stated in total-variation distance.

6 Gravitational Compatibility

Geometry, matter, and quantum fields share the one present of each causal domain of common origin [A1 and A2], which the cosmic foliation places [A6]. The adopted physical dynamics governs gravity [A5], and the theory prescribes no gravitational response to actuality beyond that dynamics.

In the classical regime the Einstein-matter equations govern, with a well-posed Cauchy problem on globally hyperbolic developments [17, 39] [Section 4.1]. In linearized quantum gravity the gravitationally mediated which-path gedankenexperiment exhibits no violation of causality or complementarity [8, 21]. Where the unitary linearized theory is the adopted weak-field dynamics, part (i) of Remark 27 makes the accessible marginals its own. Under that dynamics two masses held in superposed positions become entangled through their gravitational interaction [14, 47]. An experiment that finds no such entanglement refutes every realization that adopts that dynamics. These two regimes carry the theory's empirical gravitational claims.

Bohmian canonical geometrodynamics is the reference model for a joint realization. In it, quantum geometry and a scalar matter field undergo guided evolution under a common Wheeler-DeWitt state, with no separate actualization operation [27]. Its dynamics depends on the choice of lapse, and its law is deterministic. In the Kantowski-Sachs and $\mathbb{T}^3$ minisuperspace sectors a jump law of the actual scalar variables joins the guided evolution [Proposition 38].

A joint realization owes the theory two conditions. First, any source its gravitational field equations carry, as the Einstein equations do, is complete and conserved along each admitted continuation, not only on average. Second, the realization determines, from the complete present and the law, which domains each event of that present belongs to, consistently with present sufficiency [A2, Section 3].

6.1 The black-hole horizon

General relativity gives a black hole an apparent horizon and an event horizon, and different data fix them. The apparent horizon is the surface where the outgoing light front stops expanding: a quasi-local object on a single slice [4, 40]. In the representation, the event horizon is the boundary of the region from which no signal reaches future null infinity. So the completed development defines it. The reality criterion grounds a concrete object in present data [Section 2.3], and the question is which data fix each horizon. Ingoing Vaidya collapse, a black hole fed by infalling null dust, answers it exactly on the null slices of constant advanced time.

Proposition 31 (Apparent horizon from local data, event horizon from the future boundary)**.** *In ingoing Vaidya collapse with non-decreasing mass $M(v) > 0$, $M \in C^1$, and $M(v) \to m_f < \infty$ as $v \to +\infty$, the apparent horizon is fixed by local metric data at advanced time v: the surface $r = 2M(v)$, where the outgoing null expansion θ_+ vanishes, is the round marginally-outer-trapped surface on the null hypersurface $v = const$, with no dependence on $M'(v)$ or on M at any other advanced time, and the union of these surfaces over v is the trapping horizon of [40]. The event horizon is an object of the future boundary: it bounds the causal past of future null infinity. The equation of its null generators, $R'(v) = \frac{1}{2}(1 - 2M(v)/R)$, is repelling forward in v. Where the location $r = R(v)$ of the event horizon is a positive solution of that equation selected by the future condition $R - 2M \to 0$ as $v \to +\infty$, $R(v)$ depends on $M(v')$ for $v' \geq v$, satisfies $R(v) \geq 2M(v)$ with the excess a nonnegative integral of the future accretion rate M' weighted by a kernel, and coincides with the apparent horizon at v iff no accretion occurs after v.*

The proof is in Appendix A.8.

Remark 32 (The event horizon and the continuations of the present). The event horizon is a construct of the completed causal structure of general relativity, and no future exists to fix it [A1, Definition 9]. Its location at v is fixed only through the continuations the law admits from

the present. Where the law and the present slice's data fix the accretion at later advanced times, a determined continuation fixes it. Where they do not, it is a future contingent. The apparent horizon needs no continuation; local data at v fix it [Proposition 31]. The null hypersurfaces $v = \text{const}$ slice the Vaidya geometry, and the cosmic foliation, whose slices are spacelike Cauchy surfaces, does not contain them. The expansion of a round sphere depends on the sphere alone. So on every spherically symmetric spacelike slice the round marginally outer trapped sphere lies at $r = 2M(v)$, at the advanced time where the slice meets that surface. Local data fix it there as well.

7 Physical Realizations

The axioms fix what exists and what every dynamics of the present must meet. A realization [Definition 1] supplies the physical dynamics that meets them. This section states what every realization must supply and what it is free to choose. It then describes what is built: the family of equivariant jump laws on finite configuration spaces, the readouts and Bell correlations those laws carry, the classical Einstein-matter realization, and one law of quantum geometry and matter in the Kantowski-Sachs and $\mathbb{T}^3$ minisuperspace sectors.

7.1 Realization requirements

Quantum consistency is the requirement [A7]. A realization gives the readouts recorded in its actual physical variables the probabilities and joint correlations that the present quantum state fixes through the Born rule of the instrument [Section 5.2]. Its law of the actual physical variables is stochastic [A5] and reads nothing beyond the complete present [A4]. The realizations of this paper meet quantum consistency through two sufficient conditions, which not every realization owes. The first is quantum equilibrium, with the readouts recorded in the actual configuration [Definition 24 and Remark 28]. The second, for sequences of readouts, is a law whose flow is supported where the Hamiltonian couples, so that records persist [Lemma 29 and Proposition 33]. A joint realization owes the two conditions of Section 6. Its source is complete and conserved along each admitted continuation, and the complete present and the law fix domain membership. Every realization places its slices by a causal placement law, and every slice is a spacelike Cauchy surface of the domain [A6, Remark 19].

Realization inputs come with a realization and are not commitments of the theory. They are the data of its placement law (for the lapse driver, $K_\star$, the initial slice, and the initial lapse; Remark 19), the law adopted for the quantum state (in the unitary realization, the unitary law of $\mathbb{V}$ [Definition 52]), the law of the actual physical variables [Section 5.2], and the initial distribution of the actual physical variables.

7.2 Quantum mechanics

On a finite configuration space a jump law meets quantum consistency through equivariance [Lemma 26], and the equivariant laws at a present form an explicit family.

Proposition 33 (The equivariant jump laws at a present)**.** *On a finite configuration space of n configurations let H be Hermitian and ψ the amplitude at one present, with Born weights $P_i = |\psi_i|^2$ and current J [Definition 25]. Let a law at that present have nonnegative rates, and let its flow F_{ca} from a to c be the rate from a to c times P_a.*

(i) The master equation at P moves each Born weight P_i at the rate $\sum_j J_{ij}$ exactly when $F = \max(0, J + C) + S$ entrywise, with C antisymmetric with zero row sums (the cycle term*) and S symmetric and nonnegative (the* traffic term*). Then $C = F - F^{\mathsf{T}} - J$ and $S = \min(F, F^{\mathsf{T}})$, and at positive Born weights every such pair is the flow of a law with*

nonnegative rates. The jump law is the member $C = 0$, $S = 0$, and the law with the further rate is the member $C = 0$, $S_{ac} = \lambda_{ac}|\psi_a||\psi_c|$.

(ii) On a pair of configurations that H does not couple, a member has no flow in either direction exactly when C and S vanish on that pair.

(iii) Let G be a graph on the configurations with e edges and k connected components. Two members whose flows vanish off the edges of G differ by a divergence-free flow on its $2e$ directed edges. These flows form a space of dimension $2e - n + k$, the antisymmetric ones a subspace of dimension $e - n + k$ and the symmetric ones a subspace of dimension e. At positive Born weights and a member whose flow is positive on every edge of G and zero off them, every divergence-free flow on G is a direction along which the members continue.

The proof is in Appendix A.10. On the complete graph, with no constraint on the support, the count is $(n-1)^2$. Vanishing Born weights and flows forced to vanish remove edges and lower it. A persisting record or a conserved quantity of H leaves its value classes uncoupled. By (ii) a member that keeps it has no flow between them, so they lie in different components of the graph of its flow. The family is larger than its members with $C = 0$. On three configurations, with every off-diagonal entry of H equal to 1 and the uniform amplitude, the current vanishes. The cyclic rates $\kappa \neq 0$ from each configuration to the next are then equivariant with cycle term $\kappa/3$.

The classification is instantaneous: it fixes the equivariant rates at one present and constructs no process over an interval. The jump law and the law with the further rate are constructed over intervals, with transition probabilities between slices that compose and carry the Born weights [Lemma 26 and Propositions 46 and 48]. These laws are constructed on finite configuration spaces and, through creation and annihilation, in regularized field theory [Section 5.2]. The construction covers no interacting continuum field theory with its cutoffs removed. Within the family the choice of law leaves the Born statistics of recorded readouts unchanged. It decides which records persist and which past propositions are determinate [Lemma 29, Section 8], and regularity of the law bounds how it can do so.

Proposition 34 (Law-dependent custody)**.** *(i) Let a member on a finite configuration space under a constant Hamiltonian have cycle and traffic terms continuous along the state, and transition probabilities that, from configurations of positive weight, are stochastic, compose, and carry the Born weights. On an interval where two configurations have positive weight and continuous rates, let the probability of staying be at least that of the path with no jump, and the probability of passing from one to the other at least that of the path with one jump between them. Let the member keep a past distinction: a proposition about the configuration at an earlier slice has a truth value at every complete present of positive weight from some parameter on, and the configurations of those presents fall into two history classes by the side of the proposition on which their admitted earlier configurations lie. If its signed field $J + C$ is $2p - 1$ times continuously differentiable on every pair near every Born node, with p the greatest vanishing order of the amplitudes that vanish there, the partition into the two history classes is the same at every later present and no admitted transition crosses it. On every interval free of Born nodes from that parameter on, each history class has a Born weight independent of the parameter. A state carrying r distinct energies has $p \leq r - 1$, so a signed field $2r - 3$ times continuously differentiable along the state excludes every change of that partition.*

(ii) On four configurations in a ring, let $H_{01} = H_{23} = a > 0$, $H_{12} = H_{30} = g > 0$, the amplitude on the slice at parameter 0 be $(1, 0, 1, 0)/\sqrt{2}$, and Z the cycle current $0 \to 1 \to 2 \to 3 \to 0$. The member with cycle term $c(\psi)Z$, $c = \min(c_1, c_2)$, $c_1 = (g/a)\max(0, J_{10}) + \max(0, J_{01})$, $c_2 = (g/a)\max(0, J_{32}) + \max(0, J_{23})$, and no traffic term gives the proposition that the configuration at parameter 0 was 0 a truth value at every complete present of positive

Born weight at every later parameter. Under the jump law and under the law with the further rate the same proposition has no truth value at any such present from the parameter $\pi/(2(a+g))$ *on.*

The proof is in Appendix A.11. The member of (ii) is neither law of Definition 25. Its cycle term is built for this ring and this state and gives no rule from a Hamiltonian and a state to rates. The two laws of Definition 25 are such rules and extend to regularized field theory; this member does not. That term is continuous along the state and not differentiable at the nodes, where the partition changes. At a simple node (i) asks only one derivative. A partition that never changes is not excluded. On the ring of (ii), $\{0,1\}$ and $\{0,3\}$ each have Born weight $\frac{1}{2}$ at every parameter. The member with cycle term $(g/a)J_{10}Z$ has no flow between $\{0,1\}$ and $\{2,3\}$, and the member with $-J_{10}Z$ none between $\{0,3\}$ and $\{1,2\}$. Both terms are polynomials in the state. With the data on the slice at parameter $\pi/(4(a+g))$, where every Born weight is $\frac{1}{4}$, the first keeps the proposition that the configuration there lay in $\{0,1\}$, the second that it lay in $\{0,3\}$. By (i) no member keeps the proposition that it was 0. The Hamiltonian and the state fix the cuts a member can keep, and these are not the unions of the classes of any one relation.

7.3 Measurement and nonlocality

No distant setting changes the law of the record strings that persist to the present.

Proposition 35 (Recorded readouts)**.** *Let a finite configuration space be the product of a local factor, instrument and memory included, and a distant factor. Let the joint amplitude evolve by* $H(t) = H_A(t) \otimes I + I \otimes H_B^{\theta}(t)$*, with Hermitian terms piecewise continuous in the parameter, the local term* H_A *the same for both settings* $\theta = 0, 1$*, and the setting an exogenous index of the distant term. Let the law of the actual configuration, under each setting, give the configuration at the present the Born weights of that setting's amplitude there, and let every reading taken at an earlier parameter equal, on every path of positive weight, a function of the local configuration at the present. Then the local readouts at the present, and the record strings persisting to the present, have the same law under both settings.*

The proof is in Appendix A.12. Equivariance enters only as quantum equilibrium at the present, and persistence as a hypothesis, so the proposition holds for every law of the actual configuration that meets both. Under the jump law, records persist under the conditions of Lemma 29. A setting generator modelled as a dynamical variable belongs to the distant factor. The proposition carries the second step of Section 5.3 from one readout to the record strings of a sequence of readouts.

A Bell violation bounds from below the setting dependence of the law of the paths. Let the paths of the configuration variables of an experiment other than the setting generators and their records form a finite set Ω. Let μ_{ab} be the path law under the setting pair (a, b), $a, b \in \{0, 1\}$. A *common component* is a function ν on Ω with $0 \le \nu \le \mu_{ab}$ for all four pairs, and $1 - \sum_{\omega} \nu(\omega)$ is its *noncommon residual mass*: the path mass on which the path law depends on the setting pair.

Proposition 36 (Setting dependence of the path law)**.** *Let each* μ_{ab} *have total mass one, and let the readouts* $A_a(\omega)$ *and* $B_b(\omega)$*, with values in* $[-1, 1]$*, depend on the whole path and on their own wing's setting alone. With* $E_{ab} = \sum_{\omega} \mu_{ab}(\omega) A_a(\omega) B_b(\omega)$ *and* $S = E_{00} + E_{01} + E_{10} - E_{11}$*, every common component has noncommon residual mass at least* $(|S| - 2)/2$*. The pointwise minimum of the four path laws is the greatest common component.*

Corollary 37 (Prepared path laws)**.** *Let each path begin with the complete present* z *at the preparation, let* $\mu_{ab}(z, \omega) = p(z)\, \mu_{ab}(\omega \mid z)$ *with a preparation law* p *that does not depend on the settings, and let* S_z *be the value of* S *for the path laws* $\mu_{ab}(\cdot \mid z)$*. Then every common component has noncommon residual mass at least* $\frac{1}{2} \sum_z p(z) \big(|S_z| - 2\big)_+$*, and this is at least* $(|S| - 2)/2$*.*

The proofs are in Appendix A.13. At the Tsirelson value $|S| = 2\sqrt{2}$ the noncommon residual mass is at least $\sqrt{2} - 1$. The common component with these readouts is a local model of mass $\sum_\omega \nu(\omega)$. The proposition is therefore the bound of Abramsky, Barbosa, and Mansfield [1] in the form of a path law. That bound caps the normalized violation at the contextual fraction. It bounds the path mass on which the law depends on the setting pair. It bounds no pairing of paths across settings and no distance between their endpoint laws. The corollary can exceed the pooled bound strictly: two equally weighted preparations with opposite Tsirelson correlations give $S = 0$ and the bound $\sqrt{2} - 1$. The bound reads the continuation law of each complete present, which pooled statistics can conceal [Section 3]. Under the hypotheses of Lemma 30 the violation lies in the failure of the outcome law to factorize given the shared data. With readouts that are functions of the whole path, the jump rates carry through the joint configuration the setting dependence of the path law that the bound requires [Section 5.3].

7.4 Classical general relativity

In the classical Einstein-matter regime the realization is the Cauchy problem of general relativity on the slices [Section 4.1]. The foliation enters no field equation, so every preferred-frame bound is met identically [Section 4]. The lapse driver of Remark 19 realizes the causal placement law on homogeneous FLRW and is adopted as its realization on perturbed FLRW backgrounds. It is not the driver the theory requires. Any placement law meeting the causal clause and the Cauchy requirement of A6 serves. Its slices have constant mean curvature only in the homogeneous case, as the fixed point $K = K_\star$ [Proposition 20]. On exact homogeneous FLRW its slices persist, with the lapse between positive bounds, in every era, radiation, dust, and Λ included. Consider the perturbed developments of Remark 19 with dust or a perfect fluid of sound speed $0 < c_s^2 < 1/3$, to the future of the slice that carries their data. There the persistence of the driver's slices as spacelike Cauchy surfaces, with the lapse between positive bounds, is argued. It is not proved, since the cited stability theorems control a different slicing. The horizons of a collapsing black hole are fixed as Proposition 31 states.

7.5 Quantum gravity

The jump law that joins the guidance of the reference model of Section 6 acts in two minisuperspace sectors. Each carries two massless scalar fields ϕ_1, ϕ_2 of period 2π: the Kantowski-Sachs geometry $-N^2 d\lambda^2 + a^2 dr^2 + b^2 d\Omega^2$ with r of period 2π, and the geometry $-N^2 d\lambda^2 + a^2 dx^2 + b^2(dy^2 + dz^2)$ on $\mathbb{T}^3$. In the first let $\alpha = \ln a$ and $s = \ln(ab)$; in the second let $\alpha = \ln(a/b)$ and $s = \ln(ab^2)$. With $q = (\alpha, s, \phi_1, \phi_2)$ and lapse $N = ab^2\tilde{N}$, the Einstein-scalar equations of either sector are the Hamilton equations of $\tilde{N}C$ on the surface $C = 0$, where

$$C = \frac{p_\alpha^2 - p_s^2}{4c} + \frac{p_1^2 + p_2^2}{2\ell} + V(s) = \tfrac{1}{2} G^{ij} p_i p_j + V(s),$$

$c, \ell > 0$ are constants, $V = -c\, e^{2s}$ for Kantowski-Sachs, and $V = 0$ on $\mathbb{T}^3$. The Wheeler-DeWitt operator $\hat{C}$ replaces p_j by $-i\partial_j$. Its coefficients are constant, so it carries no ordering ambiguity.

Proposition 38 (Dynamics in two minisuperspace sectors)**.** *Let $E > 0$ and, for $k = 0, 1$, let $f_k = u_k + iv_k$ with u_k, v_k independent real solutions of $f'' = -(E^2 + 2ck^2/\ell + 4cV(s))f$, ordered so that $w_k = v_k u_k' - u_k v_k' > 0$. Let $\Psi = e^{-iE\alpha}(f_0(s) + \varepsilon e^{i\phi_1} f_1(s))$, $\Psi = Re^{iS}$, $p_s = \partial_s S$, and $j = -\operatorname{Im}(\bar{\Psi}\partial_s \Psi)/(2c)$. Then $\hat{C}\Psi = 0$, and on a compact window $s_0 \le s \le s_1$ there is $\varepsilon_* > 0$ with $j > 0$ for $|\varepsilon| < \varepsilon_*$; on $\mathbb{T}^3$ with $f_k = e^{-i\nu_k s}$, $\nu_k^2 = E^2 + 2ck^2/\ell$, this holds for every s when $0 \le \varepsilon < \nu_0/\nu_1$. For such ε, let the actual configuration follow $\dot{q}^i = \tilde{N} G^{ij}\partial_j S$ with $\tilde{N} = 2c/(-p_s)$, and at the times of a Poisson process of rate $\gamma > 0$ in λ let ϕ_2 be redrawn uniformly and ϕ_1 be redrawn among the solutions ϕ_1' of $p_s(s, \phi_1') = p_s(s, \phi_1)$ with probabilities proportional to $j/|\partial_{\phi_1} p_s|$ at ϕ_1', ϕ_1 being kept where $\partial_{\phi_1} p_s = 0$. On the window, and on $\mathbb{T}^3$ at every s:*

(i) *every configuration on every path the law admits satisfies* $C(q, \nabla S) + Q = 0$ *with* $Q = -G^{ij}\partial_i\partial_j R/(2R)$*, and a jump changes neither the geometry* (α, s) *nor its momenta* $(p_\alpha, p_s) = (-E, p_s)$*;*

(ii) *between jumps the realized geometry obeys the Einstein equations of the sector with the complete source, the scalar stress at the momenta* $\nabla_\phi S$ *together with the stress of* Q*; on every path the energy of the complete source,* $-(p_\alpha^2 - p_s^2)/(4c) - V(s)$*, is continuous, the complete source is conserved, and in the proper time of the slices* a *and* b *are continuously differentiable with bounded second derivatives, smooth between jumps;*

(iii) $s = s_0 + \lambda$ *on every path, and if* (ϕ_1, ϕ_2) *has the density* $j/\int j\, d\phi_1 d\phi_2$ *on the slice* $\lambda = 0$*, it has that density on every later slice;*

(iv) *from every present the law fixes a distribution of continuations and not a single one, and for every sufficiently small* $\varepsilon \neq 0$ *the momentum* p_s *depends on the actual value of* ϕ_1*; on* $\mathbb{T}^3$ *with* $0 < \varepsilon < \nu_0/\nu_1$ *and* $\theta = \phi_1 - (\nu_1 - \nu_0)s$*, a jump at* $\sin\theta \neq 0$ *keeps* ϕ_1 *or sends it to* $2(\nu_1 - \nu_0)s - \phi_1$*, with probability* $\frac{1}{2}$ *each,* $dp_s/d\lambda$ *takes opposite nonzero values at the two, and the two continuations, up to the next jump, differ in* α *and in the lapse at every* s *of an interval after the jump;*

(v) *the slices* $s =$ const *are compact connected spacelike Cauchy surfaces of the realized geometry over the window, each of constant mean curvature; on* $\mathbb{T}^3$ *with also* $(\nu_1 - \nu_0)\varepsilon < \nu_0(1-\varepsilon)/2$*, the realized geometry over all* s *is globally hyperbolic, its limit* $s \to -\infty$ *is a curvature singularity at finite proper time, and the comoving coordinates of every past-inextendible causal curve converge.*

The proof is in Appendix A.14.

One law governs geometry and matter. Its parts are a Wheeler-DeWitt state, with no parameter dependence and no clock field, and a law of the actual variables. That law guides geometry and matter together and lets the matter jump. By (iv) it meets the stochastic clause of A5. On $\mathbb{T}^3$ the matter's stochastic jumps reach the geometry: after a jump the two continuations carry different geometry momenta, scale factors, and lapses. The cosmic parameter is λ, represented by the geometric scalar $s = s_0 + \lambda$. The slices are the orbits of the symmetry group. By (v) they meet the order clause of A6 and are Cauchy surfaces of the realized geometry over the window, and on $\mathbb{T}^3$ over all s. The lapse $\tilde{N} = 2c/(-p_s)$ is read from the present state and configuration [A4]. By (ii) the complete source meets the first condition of Section 6. Between jumps its conservation follows from the Einstein equations and the contracted Bianchi identity. At a jump its energy is continuous and only its stress jumps, and the energy equation, which contains the stress undifferentiated, holds across the jump. Where the scalar momentum differs between the two points a jump connects, the scalar energy and the energy of Q jump by opposite amounts. The construction is an effective model with a canonical source. It is no solution of the Einstein equations with a massless scalar. The scalar enters the source through the momenta $\nabla_\phi S$ at its actual value, so the discontinuity of ϕ_1 at a jump contributes no impulsive stress. Between jumps the actual scalar obeys its field equation with the force of Q. At a jump the jump law, and no field equation, gives its dynamics. The state enters this source as it enters the law of the actual variables. Here Q is a function on the configuration space that the state fixes, evaluated at the actual configuration. The second condition, domain membership, is supplied in these sectors by the symmetry of the complete present, which the law preserves. No rule of the law supplies it. On $\mathbb{T}^3$ the past causal boundary of the realized geometry over all s is argued, and not proved, to be the connected image of the comoving torus. That makes the geometry one domain whose origin is the volume singularity [Appendix A.14]. In Kantowski-Sachs the window covers part of a domain. The construction carries no instrument and no record, since every jump redraws both scalars. Its content for quantum consistency is the equivariance (iii)

of the sector's Klein-Gordon density on the clock s. That density is a measure on the scalars (ϕ_1, ϕ_2), whose motion does not read α, with α given by the data. Because j does not depend on α, it is no normalized density on the whole slice. The construction's realization inputs are the clock s, the Klein-Gordon density in place of the density $|\Psi|^2$, the state family and its window, the jump rate γ, the resampling kernel, and the spectator field ϕ_2.

The construction relies on the symmetry exactly where the commitments of the theory bite: the choice of foliation, the causal clause of the placement law, and domain membership. Its lapse reads the universal state at the actual configuration, and on a homogeneous slice every open region carries the same data, so the causal clause holds trivially. The construction is no template beyond the symmetry. On inhomogeneous geometry a placement law whose lapse reads the quantum state meets the causal clause of A6 exactly when the lapse at each event depends only on data of the present slice in that event's causal past.

On finite configuration spaces this section establishes the family of equivariant jump laws at a present, and the jump law and the law with the further rate constructed over intervals. On the same spaces it establishes record strings that no distant setting reaches, and Bell violations that require the path law to depend on the setting pair. The lapse driver realizes the causal placement law on homogeneous FLRW and is adopted on perturbed FLRW. In the Kantowski-Sachs and $\mathbb{T}^3$ minisuperspace sectors one law of quantum geometry and matter meets the stochastic clause, the order clause of the foliation with slices that are Cauchy surfaces over the window, and the conserved complete source. On $\mathbb{T}^3$ the chance of the matter reaches the geometry. The paper constructs no dynamics meeting all the axioms jointly, with matter, quantum fields, and gravity coupled, and proves no joint consistency for one, operational no-signalling included. The obligation CP's own commitments place on it is a causal placement law for a lapse that reads the quantum state on inhomogeneous geometry.

8 Historical Truth

What makes a statement about the past true, when the past no longer exists? The theory answers with what exists: the actual present and the law, through every history that the law admits and that ends in the actual present.

Definition 39 (Present fiber)**.** Let the law's transition probabilities between complete presents at chosen slice parameters be purely atomic, as they are where the actual physical variables range over a countable configuration space and the rest of the complete present is fixed by the earlier present (a unitary state at a fixed background). Let S_{τ_0} be the slice on which the realization's data are given [Section 7.1], and let $\tau_0 < \tau_1 < \cdots < \tau_n$ be slice parameters from it to the present one. An *admitted history* is a sequence of complete presents at those parameters whose first present lies in the support of the realization's distribution on S_{τ_0} and in which the law *admits* each step: it assigns positive probability to reaching the present at τ_{i+1} from the present at τ_i [A4 and A5]. Let e send each admitted history to its last present. The *present fiber* of the actual physical reality Σ is the preimage

$$F_\Sigma = e^{-1}(\Sigma),$$

the set of admitted histories that end in the complete present of Σ, and its members are the *histories* of Σ. They are mathematical possibilities; none is an existing history. The realization's distribution on S_{τ_0} admits first presents without naming the actual one. Its support is nomic data of the realization, supplied with the law, and the law below includes it; in the unitary realization in quantum equilibrium it is the support of the Born weights of the quantum state on S_{τ_0}, which the unitary law fixes from the present state. A proposition about the past concerns the presents at given parameters, and the chosen parameters are taken to include every parameter it names.

Where e is injective, every nonempty fiber has one member, and every proposition about the past is true or false at the present by the posit of historical truth. Where several admitted histories end in one present, a proposition about the past has a truth value there exactly when they agree on it.

An ordinary sentence about the past expresses such a proposition through a present description. Its terms classify past configurations in present vocabulary, so a change of vocabulary changes which proposition the sentence expresses, never what was actual or the fiber.

Posit (Historical truth). *What has happened was actual when it happened, and nothing stores it. At a present whose fiber [Definition 39] is nonempty, a proposition about the past is true when it holds on every history of the fiber, false when it holds on none, and otherwise has no truth value at the present. Where the law's transition probabilities are atomic, at a present from which the law admits a continuation, a proposition about the future is true when it holds on every such continuation, false when it holds on none, and otherwise has no truth value. For the actual realization the posit adopts three conditions. The present that was actual on S_{τ_0} lies in the support of the realization's distribution there. The law admits each step from the present that was actual at one chosen parameter to the present that was actual at the next. The law admits a continuation from the actual present. The law gives each with probability one. The fiber of the actual present is then nonempty, and the sequence of complete presents that were actual at the chosen parameters is one of its histories.*

This is custodial truth. A proposition about the past stays true exactly as long as every history the present admits agrees on it. So the present keeps custody of a distinction, never of the past itself.

Where the realization's distribution on S_{τ_0} is purely atomic, a present's fiber is nonempty exactly when the law gives that present positive probability at its parameter. That probability is a sum, over chains from the initial support, of products of transition probabilities. A term is positive exactly when its chain lies in the fiber.

Posit (Present truthmaker). *Every truth is grounded in what exists. Present-tensed truth is grounded in the actual present physical reality with its nomic and relational structure, past-tensed truth in that present with the law it instantiates, through the present fiber, and future-tensed truth in that present with the law it instantiates, through the continuations that law admits.*

The present instantiates its nomic structure, of which the quantum state is part, and the law fixes the continuations the present admits. Neither the state nor the law is a further concrete object. The law is structure the present instantiates and is no summary of what exists.

Remark 40 (Truthmaker stance). CP grounds past truth in the complete present and the law, through the present fiber, and adds no property or fact for the past to bear. CP is not a Lucretian view, which grounds past truths in past-tensed or temporal distributional properties of present objects [16]. Such properties could change, so that truths about the past could shift [60]. Truth decay [Lemma 41] blocks that reversal. Nor is CP the presentism in which three-dimensional reality primitively changes and no past truth is determinate [22]. CP keeps every past truth on which the fiber agrees.

Lemma 41 (Truth decay)**.** *Let the law admit a transition from a complete present x at one slice parameter to a complete present y at the next, let the fiber of x be nonempty, and let P be a proposition about the presents at parameters no later than that of x.*

(i) If P is true at x, it is not false at y; if P is false at x, it is not true at y.

(ii) If P has no truth value at x, it has none at y.

(iii) If every complete present has at most one admissible predecessor at the previous parameter, P is true or false at x.

(iv) Where two distinct complete presents at one parameter, each with a nonempty fiber, both lead to one complete present at the next, the proposition that the present at that parameter was the first of them is true at the first and has no truth value at their common successor.

Proof. (i) A history of the fiber of x, extended by the transition, is a history of the fiber of y that agrees with it at every earlier parameter, so it satisfies P if every history of the fiber of x does, and fails it if none does. (ii) Two histories of the fiber of x, one satisfying P and one not, extend to y, and P depends only on parameters no later than that of x. (iii) With unique predecessors, two histories of one fiber agree at every parameter, by induction backward from the present. (iv) The extensions of a history through the first present and of one through the second both lie in the fiber of the common successor, and at that parameter they carry different presents. □

Applied step by step, (i) and (ii) hold along every finite sequence of admitted transitions. A past truth can therefore lose its truth value, but it never becomes false, and a lost truth value never returns. The fiber semantics is the adopted rule of the posit of historical truth. The jump law does not entail it, and the lemma states what follows from it.

A future-tensed proposition concerns the presents at chosen slices [Definition 39], and the posit of historical truth gives it its truth value.

Lemma 42 (Settling)**.** *Let the law admit a transition from a complete present x at one slice parameter to a complete present y at the next from which the law admits a continuation, and let F be a proposition about the presents at parameters no earlier than that of y.*

(i) If F is true at x, it is true at y; if F is false at x, it is false at y.

(ii) If F has no truth value at y, it has none at x.

(iii) Where x admits two distinct successors, each with a continuation, the proposition that the present at the next parameter is the first of them has no truth value at x and is true at the first.

Proof. A continuation from y, preceded by the transition, is a continuation from x that agrees with it at every parameter from that of y; this gives (i), and (ii) is its contrapositive. (iii) The continuations through the two successors differ at the next parameter. □

As the present becomes, propositions about the past only lose truth values and propositions about the future only gain them. Some propositions concern presents at parameters both before and after the present. Each is evaluated over every history of the fiber joined to every continuation the law admits from the common complete present. It is true when it holds on all of them, false when it holds on none, and otherwise has no truth value. Such a proposition can gain or lose its truth value as the present becomes, and it never passes from true to false or from false to true.

Corollary 43 (Joint consistency)**.** *Within a single domain, take finitely many presents that were actual in turn at chosen parameters, and form a course by joining an admitted history into the earliest, the presents that were actual from the earliest to the latest, and an admitted continuation from the latest. Every proposition that was true at any of them when it was present, and the negation of every proposition then false, hold jointly on every such course.*

Proof. By the posit of historical truth each step of the course is admitted, so, as in the proofs of Lemmas 41 and 42, its part up to each of the presents is a history of that present's fiber and its part from that present on is a continuation the law admits from it. A proposition true at that present holds on every such joined course, and one false there holds on none. □

What was determinate at one present never contradicts what was determinate at another.

Corollary 44 (Agreement with the block view)**.** *Let the block view of the same law represent the presents that were actual at the chosen parameters as one existing history and give every proposition about them the value it has on that history. At the actual present, every proposition about the past that has a truth value has the value the block view gives it, and the two differ exactly on the propositions on which the histories of the present fiber disagree.*

Proof. By the posit of historical truth the sequence of presents that were actual is a history of the fiber of the actual present. A proposition true there holds on every history of the fiber, so on that one, and a proposition false there holds on none. Where the histories of the fiber disagree the proposition has no truth value, while the block view assigns one. □

Openness of the future is relative to the proposition. A future-tensed proposition has no truth value exactly when the law admits continuations on which it holds and continuations on which it fails. A unique continuation through the slices it concerns settles every proposition about those presents, and branching leaves open only the propositions that separate the branches. An exactly conserved label shows the difference. The proposition that the label keeps its present value at a later slice is true now. A proposition about the configuration at that slice that holds on some admitted continuations and fails on others has no truth value.

Lemma 45 (Refinement)**.** *Let the transition probabilities of the law compose over intermediate slices (the Chapman-Kolmogorov law). Inserting slice parameters between those of a history leaves the truth value at the present of every proposition about the presents at the original parameters unchanged. In quantum equilibrium the distribution of the actual physical variables on every slice is the Born weight of the quantum state there, which the unitary law fixes from the present state, and each is the image of every earlier one under the transition probabilities. Taking the histories from an earlier slice, with first presents in the support of the Born weight on that slice, then also leaves the truth value at the present of every proposition about the presents at the original parameters unchanged.*

The proof is in Appendix A.15.

Fibers over any two sets of parameters from S_{τ_0} therefore agree on every proposition both can express, so one fiber serves every proposition. A conjunction is then true exactly when its conjuncts are. A disjunction need not behave so. Every history satisfies a proposition or its negation, so their disjunction is true also where the proposition has no truth value. The lemma concerns the choice of slices only [Proposition 48].

A historical proposition stays true while every admitted antecedent agrees on it. It loses its truth value as soon as the law admits a disagreeing antecedent into the same complete present, however small that antecedent's probability. Loss requires merging. Chance alone does not produce it. A law that branches without merging keeps every past proposition true or false. Under the jump law, a rate with positive integral over a step from one configuration into another suffices for merging when both configurations have nonempty fibers at the earlier slice and their outflow rates are integrable over the step. Configuration 1 of the example after Lemma 26 is such a point [Lemma 41].

The present fiber, and no surviving trace by itself, fixes whether a past proposition stays determinate. Take a photograph of a castle. "There was a castle" is true at the present only where every history of the fiber contains a castle. If the fiber also contains a history in which the photograph was fabricated and no castle stood, "there was a castle" has no truth value. Let a register be written from the system and, on every admitted history, never changed afterwards. It keeps the corresponding past proposition determinate, since every history of the fiber carries the marked value. Such persistence suffices for determinacy and is not necessary for it. Absorption with no detector and a persistent display illustrate two ways present physical data can obscure or retain a historical distinction. Surviving evidence can weaken gradually, but the theory assigns no intermediate degrees of truth. Where the fiber is finite and every history in it has positive

probability, a proposition about the past is true exactly when its probability conditioned on the complete present is one, false exactly when it is zero, and otherwise has no truth value.

The unitary law of the quantum state does not prevent the loss. In the unitary realization the state, without collapse, carries every alternative and does not say which configuration was actual. The present configuration need not fix it either. No temporal surrogate enters [Remark 16], and no variable stores a past configuration.

Example. Take the two configurations of the example after Lemma 26, under its law up to $\pi/12$, where the Born weights are $\frac{1}{4}$ and $\frac{3}{4}$. From $\pi/12$ to $\pi/3$ the Hamiltonian is $-\sigma_x$. Without a register, the jump law then moves the configuration only from 1 to 0, and the Born weight of configuration 1 falls to $(2-\sqrt{3})/4$. Configuration 1 therefore stays with probability $(2-\sqrt{3})/3$, and histories from both configurations at $\pi/12$ reach configuration 0 at $\pi/3$. At that present, the proposition that the configuration at $\pi/12$ was 1 has no truth value, although its probability given the present is $\sqrt{3}-1$. Now let an idealized unitary copy write the configuration at $\pi/12$ into a register in its ready state, and let nothing couple to the register afterwards. No jump then changes it. At the present with configuration 0 and register 1, every history of the fiber had configuration 1 at $\pi/12$, so the proposition is true. The register does not record the configuration at 0. At that present, the proposition that the configuration at parameter 0 was 0 still has no truth value, although its probability given the present is $\frac{1}{3}$.

In the unitary realization at a fixed background, the evolution of the state is invertible and does not read the configuration. There every history of the fiber carries the same past states, so propositions about past states and their Born weights are true or false at every present. The configuration history, by contrast, can lack a truth value. In the Example, the state and its Born weights at parameter 0 are determinate at the present with register 1. The configuration there has only conditional weights $\frac{1}{3}$ and $\frac{2}{3}$.

Which past propositions stay determinate has an exact criterion. A proposition about the presents at one parameter stays true or false at every later present exactly when a family of divisions of the reachable presents, agreeing with the proposition at that parameter, is preserved by every transition the law admits. Equivalently, no later present can be reached by histories from opposite sides of the proposition. Join two complete presents at chosen slice parameters, from that of the proposition on, when both have nonempty fibers and the law gives positive probability to the transition from one to the other. The proposition stays true or false at every later present exactly when it takes one value on the presents at its parameter in each class of this relation [Appendix A.5]. For analytic Hamiltonians, the static edge relation [Proposition 46] has these classes. Outside that class the edge relation need not classify.

Physical structure supplies such a division in several ways. The register of the Example above supplies one. An exactly conserved label supplies one only for histories it already distinguishes. Every outcome reached from one earlier present carries that present's value of the label. So under a law unchanged across an event, the label cannot distinguish the outcomes of an event that present left open. A record keeps a past proposition determinate only while the admitted dynamics preserves its distinction. High reliability alone does not suffice. A law with unique predecessors supplies the division, and so does one whose present fixes the earlier state. Determinacy therefore depends on the positive transition support of the adopted law.

Proposition 46 (Determinacy under an analytic Hamiltonian)**.** *On a finite configuration space let $\tau_0 < \tau_1 < \cdots < \tau_N$ be slice parameters, τ_0 that of the slice on which the data are given, let H be Hermitian with entries real analytic in the parameter on an open interval containing $\tau_0, \ldots, \tau_N$, and let the configuration on the slice at τ_0 be distributed by its Born weights. Let the configuration follow either the jump law or the law with the further rate [Definition 25]. Join $x \neq y$ by an edge when J_{xy} is not identically zero on the interval, under the jump law, and when none of H_{xy}, ψ_x, ψ_y is identically zero there, under the law with the further rate. Between the slice parameters the law has transition probabilities that compose over intermediate parameters*

from configurations of positive Born weight and carry the Born weights. With them, for $k < N$ and every set M of configurations, the proposition that the configuration at τ_k lay in M has a truth value at every complete present of positive Born weight at each of the slice parameters $\tau_k, \dots, \tau_N$ exactly when M takes one value on the configurations of positive Born weight at τ_k within each class of the edge relation.

The proof is in Appendix A.3. A single present can settle more than the classes do. In the example after Lemma 26, at a present at $\pi/12$ with configuration 0 the proposition that the configuration at parameter 0 was 0 is true, although configurations 0 and 1 form one class.

Proposition 47 (Merging under a constant Hamiltonian)**.** *On a finite configuration space let H be Hermitian and independent of the parameter, let the configuration on the slice at τ_0 be distributed by its Born weights, let it follow either the jump law or the law with the further rate [Definition 25], and join configurations by the edges of Proposition 46. There is a parameter T such that, at every complete present of positive Born weight at a slice parameter beyond T, the configurations at τ_0 that the present admits are exactly the configurations of positive Born weight at τ_0 in the class of its configuration.*

The proof is in Appendix A.4. In a finite closed system under a constant Hamiltonian past configurational truth is therefore perishable, with no intermediate case. A proposition about the configuration at an earlier slice is true or false at every later present when it takes one value within each class. Otherwise, beyond a finite parameter, it has no truth value at any present of positive Born weight in a class it divides. The classes are the same at every parameter. So every distinction about the configuration at a later slice that these laws keep at every later present is one the configuration on the slice of the data already carries. Under these laws a record keeps a distinction that slice does not carry only under a Hamiltonian that depends on the parameter, as in the register Example.

The same classes classify future-tensed propositions. For $k < m \leq N$ and a complete present of positive Born weight at τ_k, the proposition that the configuration at τ_m lies in M has a truth value at that present, under the law with the further rate, exactly when M takes one value on the configurations of positive Born weight at τ_m in the class of its configuration. Under the jump law the condition suffices and is not necessary. In the example after Lemma 26, configurations 0 and 1 form one class and both have positive Born weight at $\pi/12$. Yet at a present at 0 with configuration 1 the proposition that the configuration at $\pi/12$ is 1 is true.

Counts and timings of events between chosen slices are propositions about paths, and no phase encodes them. A pendulum returns to the same phase without storing its count. The cosmic parameter orders the advance of the present and counts nothing, and a distant clock's count depends on its start and the proper time it records. A path proposition is settled when the complete present and the law fix it, through the endpoint data and the admitted transitions, as in the two-copy Example, or through a persisting record [Section 8.1]. Path propositions the slice construction does not express are not classified by it [Remark 49].

Proposition 48 (The fiber as slice traces)**.** *On a finite configuration space let $\tau_0 < \cdots < \tau_N$ be slice parameters, τ_0 that of the slice on which the data are given, and take the complete present at each of them to be the configuration together with the quantum state, which evolves by its own law. Let H be Hermitian with entries continuous in the parameter, let the configuration on the slice at τ_0 be distributed by its Born weights, and let the slice parameters lie on a finite grid on each of whose open intervals every Born weight is positive throughout or zero throughout. Under the jump law [Definition 25] the configuration follows a jump process whose transition probabilities between the slice parameters, from configurations of positive Born weight, compose and carry the Born weights. With them, a sequence of complete presents at $\tau_0, \dots, \tau_N$ is a history of the present fiber of its last member exactly when it is the sequence of values at those parameters of a path of the process with finitely many jumps, each jump at positive rate at its parameter, whose*

configuration has positive Born weight at every parameter from τ_0 to τ_N. For H with entries real analytic on an open interval containing $\tau_0, \ldots, \tau_N$ such a grid exists, and the same holds under the law with the further rate.

The proof is in Appendix A.16.

Remark 49 (The reach of the classification). On continuous configuration spaces, and where the Born weights vanish at infinitely many parameters of a bounded window, the past propositions that keep a truth value at every later present are not classified by the Hamiltonian and its state. For a Hamiltonian that is only piecewise continuous, or where no finite grid separates the zeros of the Born weights, the fiber is not shown to be the set of slice traces of the law's paths, nor is it under the law with the further rate [Definition 25] for a non-analytic Hamiltonian. Which past propositions are determinate at the actual present of a realistic realization is not established.

Every law meeting the hypotheses of Lemma 29 gives the readouts recorded in persisting records the same Born weights, so those weights do not distinguish among such laws. Failure to predict does not establish indeterminism. The adopted law supplies it. A determined continuation is still not an existing future [Definition 9]. A tensed claim about a past valuation ("p was true yesterday") is itself past-tensed and, like every truth, is evaluated at the actual present.

8.1 Records

Remark 50 (Records are derivative). Records and outcomes are physical features of the actual physical reality. They carry information about what was. They are neither a stored past nor a gate for actuality. Past actuality does not require a presently accessible record.

Records are evidence of what was. They do not guarantee historical truth a priori. Imperfect records of a single outcome can disagree, and conflicting records do not by themselves establish which, if either, is correct. A record's reliability in identifying an antecedent depends on the preparation, the process that forms the record, and the joint statistics of the readouts. Born weights alone do not settle historical origin. A fabricated photograph establishes the existence of the photograph. It does not establish the occurrence of the event it depicts.

The framework fixes the definition of the present fiber; the complete actual present and the governing law fix the particular fiber. Neither a priori reasoning nor what an observer happens to read from the records determines it. Evidence and truth therefore come apart. Records provide empirical evidence about the past, while the present fiber determines which past propositions are true, which are false, and which have no truth value. A past proposition can lack a truth value when no persistent feature of the present records it. A proposition about a measured system's earlier configuration under the jump law is one.

Example. Let two copies of the witness of Lemma 26, A and B, evolve side by side, with the product of their states and the sum of their Hamiltonians. Each copy then jumps at the rates of the lemma whatever the other's configuration, so the two copies jump independently. At the parameter $-\pi/4$ each state is $(1, 0)$, so take the data on that slice, where both configurations are 0, and slice parameters $-\pi/4 < s < T < \pi/4$. On this interval each copy jumps from 0 to 1 at most once and never back, and from 0 it has jumped by t with probability $(1 + \sin 2t)/2$. At a present at T with both configurations 1, every history of the fiber has both configurations 0 at $-\pi/4$ and 1 at T. So the proposition that both copies changed configuration between those parameters is true. The fiber also contains histories in which, at s, A had jumped and B had not, and histories in which B had jumped and A had not. So neither of these propositions has a truth value [Definition 39]. The two changes happened in one order as the present became. The present at T settles that both happened and carries no record of their order [Remark 50].

Remark 51 (The two seals). What occurred was actual when it occurred and cannot be undone, although that earlier present no longer exists. The past cannot be changed, re-entered, or rewritten.

It cannot be changed because, under A1, no past exists as a target of change. It cannot be re-entered because the cosmic parameter strictly increases as the present becomes, and no earlier slice becomes actual again [A6, Remark 21]. It cannot be rewritten in the sense of making a past truth false: under the conditions of Lemma 41, no past truth becomes false.

A past truth can lose its truth value, but this does not undo what was actual when it happened. The seal is general: at every present, whatever happened was actual. A particular proposition about what happened is a proposition about the past like any other, settled by the present fiber, and where the fiber disagrees it has no truth value. Actuality does not wait on the present to store it.

9 Established Results and Their Scope

9.1 Operational standing

Definition 52 (The base structure)**.** The base structure $\mathbb{V}$ is unitary quantum mechanics with the Born rule, taken as a formal object with no position on temporal ontology. The Born weights [Definition 23] are content of $\mathbb{V}$.

Consider an ontology that assigns the quantum state of $\mathbb{V}$ under its ordinary law and applies no operation to it beyond a common protocol. It assigns every accessible algebra the same restriction of that state, and so the same accessible marginals. CP is such an ontology wherever its unitary realization supplies the law of $\mathbb{V}$ [Remark 28]. The block, the whole four-dimensional structure under unitary dynamics, is another.

Table 1: Operational sameness, existential difference: what $\mathbb{V}$ and CP each fix.

What is asked of the theory	**$\mathbb{V}$ (base structure)**	**CP**
Accessible marginals of the state	fixed	fixed, at every time, where the state's law is that of $\mathbb{V}$ (Remark 27(i))
Distribution of the designated actual variables and its coarse-grained readouts	Born statistics	the Born distribution, as the equilibrium measure, where the state's law is that of $\mathbb{V}$ and the realization is in quantum equilibrium (Remark 27(ii), Remark 28)
That an outcome is actual now	the base structure alone singles out no privileged actual, becoming present	one physical reality is actual [A1]
A present that becomes	no fact of becoming	the present intrinsically becomes [A3]
Objects defined by the completed causal structure	no position on temporal ontology	the event horizon is fixed only through the continuations the law admits from the present [Remark 32]
Whether the past is gone or remains	no position on temporal ontology	it is gone; what is true of it is fixed by the present and the law, under the posit of historical truth

Remark 53 (Which ontology obtains is not an observable). No self-adjoint operator of an accessible algebra represents which of these ontologies obtains. Actuality is an ontological

predicate of the present physical reality, not a measurable magnitude with a spectrum. For every measurement on an accessible algebra, the Born statistics are a functional of the restriction of the state, which the ontologies share. Relative to those algebras, becoming is therefore operationally undecidable among them. The choice turns on which ontology grounds actuality, becoming, and present-tensed truth. No operational result settles or justifies that choice. The one present becomes as a whole [Remark 11]. Its becoming is no influence propagating among its events at a finite superluminal speed. On that account, influences reproducing quantum correlations permit superluminal signalling [6].

9.2 Where the theory competes

The difference between the theory and $\mathbb{V}$ is ontological.

Table 2: Commitments at a glance. Each row carries the status of its governing statement.

Structure asserted	**What the theory commits to**
Which distant events share the present	a posited cosmic foliation (axiom, A6)
A past truth that never turns false	the posit of the present truthmaker and the posit of historical truth; no past truth becomes false [Lemma 41]
Single-history quantum mechanics	one actual physical reality (axiom, A1)
A future that does not exist	one present only (axiom, A1)
A future open where the law admits alternatives	a stochastic law of the actual physical variables (axiom, A5)
Born weights	fixed by the present quantum state through the Born rule of the realization's instrument, with agreement of the readout statistics with those measured as the test of a realization (axiom, A7); in the unitary realization, those of $\mathbb{V}$ [Definition 52]
Algebraic locality	a factored theorem at a fixed background: Einstein locality gives commutation, and commutation with completeness gives nonselective preservation [Lemma 54 and Corollary 57]

9.3 Parsimony on two axes

Parsimony asks two things of a theory. It identifies what it takes to be actual, rather than leaving that question to its mathematical representation. Each of its posits does work: removing the posit changes a prediction, leaves a law of the theory undefined, or leaves a class of its truths ungrounded.

Removal tests identify the work done by CP's commitments. Without the posit of the present truthmaker, presentism remains, but tensed truths can be left ungrounded. Without the cosmic foliation, univocity still requires one unfragmented present [Remark 12]. The theory then no longer fixes which distant events share that present, or where it is situated in the realized geometry [A6]. The foliation carries no stress-energy and is no further field [Remark 19]. So its removal changes no classical equation and, where the quantum state obeys the law of $\mathbb{V}$, no state marginal [Remark 27 (i)]. It leaves the readout law undefined: the unitary realization poses the law of its actual variables on the slices [Section 5.2]. Physical evolution governs how the complete present changes; the cosmic foliation determines the placement of its representations.

9.4 Becoming without a new prediction

Why introduce intrinsic becoming if it predicts nothing different? On the accessible algebras the unitary realization shares every marginal with each ontology that assigns the same quantum

state, the block among them [Sections 9.1 and 9.2]. Empirical equivalence does not establish ontological equivalence.

Where their predictions coincide, CP and the block view share the mathematical solution and disagree about what it represents as real [Remark 16]. The ontological disagreement extends to which past propositions have truth values. The theory agrees with the block view of the same law wherever it assigns a value. It differs exactly where the histories of the present fiber disagree [Corollary 44]. The theory keeps the mathematics of the regimes it adopts (general relativity and, in the unitary realization, unitary quantum mechanics). It states its own ontological commitment as a posit [A3]. Remove intrinsic becoming while retaining the ordered foliation and the designation of one slice as actual. If that designation remains fixed, the present does not become; if it advances, intrinsic becoming has been reintroduced. The posit contributes the claim that the present becomes in the order the foliation represents, not a new physical mechanism or prediction.

What the theory holds to exist is one present within each domain, shared by every real event of it [Remark 11], and that present intrinsically becomes [A3]. Outcomes and records are features of it, definite in the actual physical variables, a definiteness that requires no collapse [Section 5 and Remark 50]. The future is not actual, whether or not the law fixes it [Definition 9]. On this ontology experienced passage has a counterpart in what exists: the one present that becomes. That reading interprets the commitments and is no evidence for them.

9.5 What the framework contributes

The framework contributes an explicit axiomatic formulation of presentism: seven axioms, two posits on truth, and the low-entropy premise [Remark 21]. It places univocity at the existence root, with the slice of the now downstream [Remarks 5 and 11]. It adopts as a posit the grounding of historical truth in the present and the law [46]. Truth decay follows from that posit's semantics of the present fiber, and the law does not yield it [Lemma 41]. Its cosmic foliation carries a causal placement law. The lapse driver is an adopted realization of that law on perturbed FLRW and does not derive the foliation [Remark 19].

10 Conclusion

CP posits one intrinsically becoming actual present within each causal domain of common origin. The cosmic foliation orders its advance. The future does not exist. Quantum alternatives are possibilities, none of them a parallel actuality [A1 to A3, A5 and A6].

Single outcomes require no collapse [Section 5]. In quantum equilibrium and at a fixed background, the unitary realization preserves the observable marginal probabilities of unitary quantum mechanics. Independently produced settings there do not signal through local readouts [Remarks 27 and 28 and Section 5.3]. The jump law keeps the records of readout sequences on finite configuration spaces, for piecewise continuous Hamiltonians whose Born weights have finitely many isolated zeros. These records therefore carry their Born weights [Lemma 29].

What occurred cannot be undone or re-entered [Remark 51]. Past truth is custodial. It can lose its truth value and never reverses, and what was determinate at different presents is jointly consistent [Lemma 41 and Corollary 43]. Under a constant Hamiltonian and either law of Definition 25, finite closed systems lose past configurational truth except across cuts no admitted transition crosses [Proposition 47]. CP's contribution is an explicit axiomatic formulation that distinguishes what the laws permit from what actually exists, even where observations cannot distinguish them. The formulation states one present that becomes, with definite records that require no collapse, and a future that is not actual [Section 9.4].

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A Proofs

A.1 Operational no-signalling at a fixed background

Lemma 54 (Commutation from Einstein locality)**.** *Let $\mathcal{A}$ be a net of local observable algebras satisfying Einstein locality: for spacelike-separated regions O_A and O_B, $\mathcal{A}(O_A) \subseteq \mathcal{A}(O_B)'$. Then every $M \in \mathcal{A}(O_A)$ commutes with every effect $E \in \mathcal{A}(O_B)$.*

Proof. $M \in \mathcal{A}(O_A) \subseteq \mathcal{A}(O_B)'$, the commutant of $\mathcal{A}(O_B)$, so M commutes with every element of $\mathcal{A}(O_B)$, E included. □

Proposition 55 (Operational no-signalling from completeness and spacelike commutation)**.** *Let φ be a state on the observable algebra, let Alice's setting θ index a finite family of measurement operators $\{M_k^\theta\}$ localized in her region, and let E be an effect localized in a region spacelike to hers. Assume (i) completeness, $\sum_k (M_k^\theta)^\dagger M_k^\theta = I$ for every setting, and (ii) that each M_k^θ commutes with E. Then*

$$\sum_k \varphi\big((M_k^\theta)^\dagger E\, M_k^\theta\big) \;=\; \varphi(E)$$

for every setting θ and every such E.

Proof. By (ii), $(M_k^\theta)^\dagger E\, M_k^\theta = (M_k^\theta)^\dagger M_k^\theta E$ for each k. Summing over k and applying (i) gives $\sum_k (M_k^\theta)^\dagger E\, M_k^\theta = E$, and evaluating φ on both sides gives the identity, whose right-hand side carries no θ. □

The identity is the textbook no-signalling property of a complete Kraus family commuting with the distant effect. It uses no idempotence, orthogonality, self-adjointness, or projectivity of the family. Lemma 54 supplies hypothesis (ii) whenever the family is localized as Einstein locality requires. Remark 58 shows that the hypothesis cannot be dropped.

Corollary 56 (Nonselective locality, composed)**.** *Let $\{M_k^\theta\} \subset \mathcal{A}(O_A)$ be finite and complete for every setting θ, with O_A spacelike to O_B and $\mathcal{A}(O_A) \subseteq \mathcal{A}(O_B)'$, and let every outcome have positive weight, $\varphi((M_k^\theta)^\dagger M_k^\theta) > 0$. Then for every $E \in \mathcal{A}(O_B)$ and every θ, the average of the remote values conditioned on each outcome, $\varphi((M_k^\theta)^\dagger E\, M_k^\theta)/\varphi((M_k^\theta)^\dagger M_k^\theta)$, weighted by $\varphi((M_k^\theta)^\dagger M_k^\theta)$, equals $\varphi(E)$.*

Proof. The weighted average is $\sum_k \varphi((M_k^\theta)^\dagger E\, M_k^\theta)$, and Lemma 54 and Proposition 55 give $\varphi(E)$. □

Corollary 57 (Setting no-signalling at a fixed background)**.** *Let ω be a state on the quasi-local field algebra $\mathcal{A}$ at a fixed background, let O_A and O_B be spacelike-separated regions with $\mathcal{A}(O_A) \subseteq \mathcal{A}(O_B)'$, and let Alice's setting θ index any finite normalized local family $\{M_k^\theta\} \subset \mathcal{A}(O_A)$. Then $\sum_k \omega((M_k^\theta)^\dagger E\, M_k^\theta) = \omega(E)$ for every $E \in \mathcal{A}(O_B)$, with no dependence on θ.*

Proof. Lemma 54 supplies hypothesis (ii) of Proposition 55, and normalization is hypothesis (i). □

Remark 58 (Ontic nonlocality and operational signalling)*.* Proposition 55 constrains the unconditioned average and nothing else. The actual physical variables can depend on the setting, and the distant statistics conditioned on Alice's outcome can depend on both that outcome and that setting: an individual conditioned remote value $\varphi((M_k^\theta)^\dagger E\, M_k^\theta)/\varphi((M_k^\theta)^\dagger M_k^\theta)$ can differ from $\varphi(E)$, setting by setting and outcome by outcome. The average over her outcomes does not depend on the setting. A distant experimenter without access to her outcome has that average. That separation is the distinction between ontic nonlocality and operational signalling. Dropping commutation admits a signalling countermodel: at $\varphi = |0\rangle\langle 0|$ and the effect $E = \frac{1}{2}(I + Z)$, a Z-basis setting returns $\sum_k \varphi(P_k E P_k) = 1$ and an X-basis setting returns $\frac{1}{2}$. Without completeness the family is a postselected operation, and the identity holds for the complete instrument, its failure outcome included.

A.2 Born equivariance of the jump process

Proof of Lemma 26. Hermiticity gives $J_{ji} = -J_{ij}$, and $J_{ij} = 0$ where $\psi_j = 0$. Gain minus loss at configuration i is then $\sum_j \big(\max(0, J_{ij}) - \max(0, -J_{ij})\big) = \sum_j J_{ij}$, and the Schrödinger equation gives $\frac{d}{dt}|\psi_i|^2 = 2\,\mathrm{Re}(\bar{\psi}_i \dot{\psi}_i) = \sum_j J_{ij}$. The master equation is linear with coefficients integrable on the interval, so its solution with given initial data is unique there. For $H = \sigma_x$, $\psi_0(t) = (\cos t - \sin t)/\sqrt{2}$ and $\psi_1(t) = -i(\cos t + \sin t)/\sqrt{2}$, so $J_{10} = \cos 2t$, the reverse rate $\max(0, -\cos 2t)/|\psi_1|^2$ vanishes for $t < \pi/4$, and the rate out of 0 is $J_{10}/P_0 = -\dot{P}_0/P_0$. The probability S of no jump by t obeys $\dot{S} = (\dot{P}_0/P_0)S$ with $S(0) = 1$, so $S = P_0(t)/P_0(0) = (\cos t - \sin t)^2 = 1 - \sin 2t$. □

A.3 Determinacy under an analytic Hamiltonian

Proof of Proposition 46. Write $\rho_x = |\psi_x|^2$, $P_{s,t}(a, b)$ for the transition probabilities, $K_i = P_{\tau_i, \tau_{i+1}}$, and $a \sim b$ for the equivalence generated by the edges.

Analytic structure. The state solves a linear equation whose coefficients are real analytic on the interval, so each ψ_x and each J_{xy} is real analytic there. Such a function that is not identically zero on the interval has finitely many zeros on every compact subinterval and is nonzero somewhere in every open subinterval. So the edges do not depend on the window, and each $[\tau_i, \tau_{i+1}]$ contains finitely many zeros of the amplitudes that are not identically zero. These zeros are the node times.

Rates between node times. On an open interval free of node times each ρ_x is positive throughout or ψ_x vanishes identically, the rates are nonnegative and continuous, and the Born weights solve the master equation [Lemma 26]; the added rates carry no net flow, since $\rho_a \lambda_{ac} |\psi_a||\psi_c|/\rho_a$ is symmetric in a and c. The flows $\rho_b\, \sigma(b \to c)$, equal to $\max(0, J_{cb})$ plus $\lambda_{bc}|\psi_b||\psi_c|$, are continuous, so a constant B bounds them on the interval. A positive rate from a to $c \neq a$ lies on an edge.

Transition probabilities between node times. Let (s, u) contain no node time. For $s < t \leq t' < u$ the master equation has a unique solution from each configuration; these give stochastic $P_{t,t'}$ that compose and carry ρ, and each row stays in the class of its start, since no flow enters the complement of a class. For $\rho_x(s) > 0$ the bounds $\rho_x(r)(1 - P_{r,r'}(x, x)) \leq nB(r' - r)$ and $\frac{1}{2}\sum_c |P_{r,t}(x, c) - P_{r',t}(x, c)| \leq 1 - P_{r,r'}(x, x)$ make $P_{r,t}(x, \cdot)$ converge as r decreases to s. The limit $P_{s,t}(x, \cdot)$ is stochastic, composes, carries $\rho(s)$, and tends to the unit row at x as t decreases

to s. For $\rho_a(t) > 0$, the bound $\rho_a(t)P_{t,r}(a,\cdot) \le \rho(r)$ and the flow bound make $P_{t,r}(a,\cdot)$ Lipschitz in r on $[t,u)$, and its limit $P_{t,u}(a,\cdot)$ is stochastic, composes, and carries ρ. Joining the two pieces at an interior parameter gives transition probabilities on $[s,u]$ from every configuration of positive weight, and their positive entries stay in classes.

Transition probabilities between slices. Joining these at the node times of $[\tau_i, \tau_{i+1}]$ gives K_i. Composition and carriage hold from configurations of positive weight, and these reach only configurations of positive weight. The histories start in the configurations of positive weight at τ_0. So (a) $\rho_b(\tau_{i+1}) > 0$ exactly when $\rho_a(\tau_i) > 0$ and $K_i(a,b) > 0$ for some a, and by induction a present has a nonempty fiber at τ_i exactly when its Born weight there is positive; and (b) $\rho_a(\tau_i) > 0$ and $K_i(a,b) > 0$ give $a \sim b$.

Sufficiency. Let M take one value on the configurations of positive weight at τ_k within each class, and let S hold at c when some $a \in M$ with $\rho_a(\tau_k) > 0$ has $a \sim c$. Then S agrees with M on the configurations of positive weight at τ_k, and by (b) every admitted step from a present with nonempty fiber preserves S. Every history of the fiber of a present at τ_j, $k \le j \le N$, therefore has $h_k \in M$ exactly when S holds at that present, so the histories agree.

Necessity. Let the proposition have a truth value at every present of positive weight at $\tau_k, \dots, \tau_N$. Then no d has $K_k(b,d) > 0$ and $K_k(c,d) > 0$ with $b \in M$ and $c \notin M$ both of positive weight at τ_k, since histories through b and through c would share the present d. Choose $w \in (\tau_k, \tau_{k+1}]$ with no node time in (τ_k, w) and $t_0 \in (\tau_k, w)$, and let A be the set of x with $P_{\tau_k,t_0}(b,x) > 0$ for some $b \in M$ of positive weight at τ_k. Each such b lies in A, by right continuity at τ_k and the positive probability of staying on (τ_k, w). Let $x \in A$, reached from b, and let $\{x,y\}$ be an edge. Then $\rho_y(t_0) > 0$, and by carriage $P_{\tau_k,t_0}(c,y) > 0$ for some c of positive weight at τ_k. If $c \notin M$, some $r \in (t_0, w)$ carries a positive rate between x and y, say from x to y: staying carries the occupation of x from b to r, the rate gives $P_{r,t'}(x,y) > 0$ for some $t' \in (r,w)$, and staying carries the occupation of y from c to t'. The row of y at t' reaches some d at τ_{k+1}, and composition gives $K_k(b,d) > 0$ and $K_k(c,d) > 0$, which is excluded; the other direction is symmetric. So $c \in M$ and $y \in A$. Along a path of edges, every a' of positive weight at τ_k in the class of some $a \in M$ of positive weight lies in A: some $b \in M$ has $P_{\tau_k,t_0}(b,a') > 0$, and $P_{\tau_k,t_0}(a',a') > 0$, so b and a' reach a common d at τ_{k+1} and $a' \in M$.

The argument uses only the properties established for the transition probabilities, so the verdict holds for every family with them.

The future. Let x have positive weight at τ_k and $k < m \le N$. By (a) and (b), every continuation of x has at τ_m a configuration of positive weight in the class of x, so a set M taking one value on these gives every continuation the same verdict, under either law. Under the law with the further rate every edge carries a positive rate in each direction at every parameter at which both its ends have positive weight, since $\lambda_{ac}|\psi_a||\psi_c|/\rho_a > 0$ there. Let z have positive weight at τ_{i+1} in the class of x, of positive weight at τ_i. By carriage some a of positive weight at τ_i has $K_i(a,z) > 0$, and $a \sim x$ by (b). Choose $w \in (\tau_i, \tau_{i+1}]$ with no node time in (τ_i, w); there every configuration of the class has positive weight, so jumps along a path of edges from x to a, with staying between them, give $P_{\tau_i,t}(x,a) > 0$ for $t \in (\tau_i, w)$. Since $P_{\tau_i,t}(a,a)$ tends to 1 as t decreases to τ_i, some such t has $P_{t,\tau_{i+1}}(a,z) > 0$, and composition gives $K_i(x,z) > 0$. By induction over the slices, every configuration of positive weight at τ_m in the class of x lies on a continuation of x, which stochastic rows extend to τ_N; so where M splits these configurations the proposition holds on some continuations and fails on others. Under the jump law the example after Lemma 26 shows the converse fails: the rate from 1 to 0 vanishes, so every continuation of configuration 1 at 0 has configuration 1 at $\pi/12$. □

A.4 Merging under a constant Hamiltonian

Proof of Proposition 47. Write $\rho_x = |\psi_x|^2$, $h = \max_{x \ne y}|H_{xy}|$, X for the configurations whose amplitude is not identically zero, and $P_{s,t}$ for the transition probabilities of Appendix A.3. A constant Hamiltonian is analytic on every interval, so they exist on every bounded interval from

τ_0 and join by composition; from configurations of positive weight they are stochastic, compose, carry ρ, and their positive entries stay in classes. For a class C let S_C be its configurations of positive weight at τ_0, and N the number of configurations of positive weight at τ_0. By composition and Lemma 45, a complete present of positive weight with configuration x at t admits configuration i at τ_0 exactly when $\rho_i(\tau_0)P_{\tau_0,t}(i,x) > 0$; write $F_t(x)$ for the set of these i. No current and no jump crosses a class boundary, so each class keeps its weight.

A nondecreasing count. Let $V(t) = \sum_x \rho_x(t)|F_t(x)|$. For $t \le t'$, carriage gives $V(t') - V(t) = \sum_{x,y} \rho_x(t)P_{t,t'}(x,y)(|F_{t'}(y)| - |F_t(x)|)$, and $P_{t,t'}(x,y) > 0$ gives $F_t(x) \subseteq F_{t'}(y)$, so every term is nonnegative. Thus V is nondecreasing, $V(\tau_0) = 1$, and $V \le N$.

Good windows. Expanding ψ in eigenvectors of H, each ρ_x and each J_{xy} is a finite sum of terms $e^{-i(E-E')t}$, hence real analytic on the line, and those not identically zero have isolated zeros. So some $s \ge \tau_0$ has $\rho_x(s) > 0$ for every $x \in X$ and, under the jump law, $J_{xy}(s) \ne 0$ on every edge; let $2b$ and $2j$ be the least of these values. Three bounds follow from $\|\psi\| = 1$ and $|e^{i\theta} - 1| \le |\theta|$: $\|\psi(t) - \psi(t')\| \le \|H\||t - t'|$; with $E_1, \dots, E_r$ the distinct energies present in ψ, $\|\psi(t+T) - e^{-iE_1T}\psi(t)\| \le d(T) = \max_m |e^{-i(E_m - E_1)T} - 1|$; and for unit vectors with $\|p - e^{i\varphi}p'\| \le \eta$, $|\rho_x - \rho'_x| \le 2\eta$ and $|J_{xy} - J'_{xy}| \le 4|H_{xy}|\eta$, both invariant under the phase. Choose $\ell > 0$ with $4\|H\|\ell \le b$ and $8h\|H\|\ell \le j$, and put $\varepsilon = \min(b/4,\, j/(8h))$ (under the law with the further rate, the conditions on j are dropped). Whenever $d(T) \le \varepsilon$, on $[s+T, s+T+\ell]$ every $x \in X$ has $\rho_x \ge b$ and, under the jump law, every edge current keeps one sign with $|J_{xy}| \ge j$; no node time lies in the window.

Rates on a window. There, under the jump law the rate from x to y along the current is $J_{yx}/\rho_x \ge j$; under the law with the further rate it is at least $\lambda_{xy}|\psi_y|/|\psi_x| \ge \lambda_{\min}\sqrt{b}$ in both directions on every edge. Call this lower bound q. The flow out of x is at most $(2\|H\| + \|\lambda\|)|\psi_x|$ (each row of H or λ has Euclidean norm at most its operator norm), so the total exit rate is at most $\Lambda = (2\|H\| + \|\lambda\|)/\sqrt{b}$, with $\lambda = 0$ under the jump law. On $[u, u+\ell]$, $K(t) = P_{u,t}$ solves $\partial_t K = K(R - D)$, $K(u) = I$, with R the rates and D the diagonal of exit rates; it is the unique solution of $K(t) = E(u,t) + \int_u^t K(\sigma)R(\sigma)E(\sigma,t)\,d\sigma$, $E(\sigma,t) = \operatorname{diag} e^{-\int_\sigma^t D}$, and the Picard iterates from E are nonnegative and increase. Hence $K(y,y) \ge e^{-\Lambda\ell}$ and, along a rate at least q, $K(x,y) \ge q\ell e^{-\Lambda\ell}$.

An increment. If at u two configurations joined by an edge, with rate at least q from x to y, have $F_u(x) \ne F_u(y)$, then $F_{u+\ell}(y) \supseteq F_u(x) \cup F_u(y)$. Either $F_u(x) \not\subseteq F_u(y)$, and the term of y staying raises V by at least $be^{-\Lambda\ell}$, or $F_u(y) \not\subseteq F_u(x)$, and the term of the jump raises it by at least $bq\ell e^{-\Lambda\ell}$. So V rises over the window by at least $\delta = be^{-\Lambda\ell}\min(1, q\ell)$.

Returning windows. Let $m = \lfloor (N-1)/\delta \rfloor + 1$ and $Q = \lceil 2\pi(m-1)/\varepsilon \rceil$. Of the $Q^{r-1} + 1$ points $\big(\{k(E_n - E_1)\ell/2\pi\}\big)_{n \le r}$, $k = 0, \dots, Q^{r-1}$, in the Q^{r-1} cubes of side $1/Q$, two share a cube, so some $1 \le k_0 \le Q^{r-1}$ gives $T^* = k_0\ell$ with $d(T^*) < 2\pi/Q$; since $|z^k - 1| \le k|z - 1|$ on the unit circle, $d(kT^*) \le \varepsilon$ for $k < m$. The windows $[s + kT^*, s + kT^* + \ell]$, $k < m$, are disjoint and satisfy the bounds above.

Merging. Were an edge to disagree at the start of each of these m windows, V would exceed $1 + m\delta > N$. So at the start t_a of one of them the fibers agree along every edge, hence are constant on each class. Every configuration of X has positive weight at t_a, and the row of each $i \in S_C$ stays in C, so the common value on C is S_C. At $t \ge t_a$, a present of positive weight with configuration $y \in C$ has as fiber the union of $F_{t_a}(x)$ over the $x \in C$ of positive weight with $P_{t_a,t}(x,y) > 0$, which is S_C. Take $T = t_a \le s + (m-1)\ell Q^{r-1}$. The configuration on any later slice is distributed by its Born weights, so the same holds with that slice in place of τ_0. □

A.5 Transition classes

Proof of the transition-class criterion. Let P be the proposition that the present at τ_k lay in a set M, and for a complete present z with nonempty fiber at a slice parameter from τ_k on let $O(z)$ be the set of presents at τ_k through which the histories of its fiber pass. A transition of

positive probability from z to z' gives $O(z) \subseteq O(z')$, since a history of the fiber of z extended by the transition is a history of the fiber of z', as in the proof of Lemma 41; and every member of $O(z)$ lies in the class of z, since a history joins it to z by transitions of positive probability. P has a truth value at z exactly when $O(z)$ lies in M or in its complement. If P takes one value on the presents at τ_k in each class, every $O(z)$ therefore lies on one side. Conversely, let every $O(z)$ lie on one side. Each $O(z)$ is nonempty, so an inclusion $O(z) \subseteq O(z')$ puts both on the same side, and the side is constant on each class. At τ_k, where $O(z) = \{z\}$, P takes one value on each class. □

For an analytic Hamiltonian this criterion and Proposition 46 both hold for every set M. Taking M the configurations of positive weight at τ_k in one class of either relation shows that, on those configurations, every class of the one lies in a class of the other, so the two agree there.

Outside the analytic class the edge relation does not classify. On three configurations let the data at 0 be $\psi = (\sqrt{p}, \sqrt{1-p}, 0)$, $0 < p < 1$, Born distributed. Let $H = \frac{\pi}{2}\sigma_x$ on configurations 1 and 3 for parameters in $[0, 1)$ and on configurations 1 and 2 for parameters in $[1, 2]$, and take the slice parameters $0, 1, 2$. On $[0, 1)$, $\psi_1 = \sqrt{p}\cos(\pi t/2)$, $\psi_3 = -i\sqrt{p}\sin(\pi t/2)$, and $J_{31} = \frac{\pi p}{2}\sin \pi t > 0$, so the jump law moves the configuration only from 1 to 3, configuration 1 stays with probability $\cos^2(\pi t/2)$, which vanishes at 1, and configuration 2 does not move. On $[1, 2]$ the same holds with 2 draining into 1 and 3 fixed. Every path from 1 at 0 is at 3 at 1 and at 2, and every path from 2 is at 2 and then at 1. The proposition that the configuration at parameter 0 was 1 is therefore true or false at every present at 1 and at 2. The currents J_{31} and J_{12} are not identically zero on $[0, 2]$, so the edge relation joins all three configurations, and the proposition takes two values on the configurations of positive weight at 0 within that one class.

A.6 Born frequencies in persisting records

Proof of Lemma 29. (i) On a history of positive weight every factor $T_j(x_j, x_{j+1})$ is positive, so each record takes at its write parameter the value it takes at t_N, and an event of the records has the weight of the same event of the configuration at t_N. Summing the history weights over $x_0, \dots, x_{N-1}$ and using $\sum_a \rho_j(a)T_j(a, c) = \rho_{j+1}(c)$ step by step gives the configuration at t_N the distribution ρ_N. Under the product hypothesis the record values are independent with distribution w, so the indicators of A are independent with mean $\sum_{a\in A} w(a)$, and Hoeffding's inequality [41, Theorem 2], applied to each tail, bounds the weight of a deviation of at least ε by $2e^{-2m\varepsilon^2}$. (ii) By (i) the two weights are $\|\Pi'_o \Pi_y V U \psi\|^2$ and $\|\Pi_y V U \psi\|^2$. Moving Π'_o and Π_y through V and Π_y through U, and using that V and U preserve the norm, gives $\|\Pi'_o U \Pi_y \psi\|^2$ and $\|\Pi_y \psi\|^2$. (iii) Write $D_k = 1_A(R_k) - p_k$. Given the earlier records, D_k has mean zero and takes values in an interval of length one; the argument needs no independence of the outcomes, only that p_k is their conditional weight given the earlier records. Hoeffding's lemma [41, eq. (4.16)], applied conditionally, bounds the conditional mean of e^{sD_k} by $e^{s^2/8}$ for every real s. Conditioning on the records one at a time from the last bounds the weighted mean of $e^{s\sum_k D_k}$ over histories by $e^{ms^2/8}$, and the exponential Chebyshev bound with $s = \pm 4\varepsilon$ on each tail gives $2e^{-2m\varepsilon^2}$. Summing (ii) over $o \in A$ gives the weight of the histories with earlier records y and $R_k \in A$ as $\|\Pi'_A U_k \Pi_y \psi_k\|^2$, and summing over all o gives the weight of y as $\|U_k \Pi_y \psi_k\|^2$, which is positive for a string of positive weight.

For the records of the jump law [Definition 25]: the rate from j to i is $\max(0, J_{ij})/|\psi_j|^2$ with $J_{ij} = 2\,\mathrm{Im}(\bar{\psi}_i H_{ij} \psi_j)$, which vanishes when $H_{ij} = 0$. So no jump of positive rate after t_i joins two values of R, every transition of positive probability from a configuration of positive Born weight preserves R, and through a Born node the same holds for the one-sided limits of the transition probabilities. Summing $\dot{P}_i = \sum_j J_{ij}$ over the configurations with one value of R cancels the currents between them and leaves no current to the other values, so the Born weight of each value is constant. The proof of (i) to (iii) uses the record property only on histories of positive weight, so its conclusions hold for R. For a Hamiltonian of this class, the rates of the jump law

are continuous and nonnegative on each open grid interval from configurations of positive weight, and the jump law has transition probabilities on a grid through the interval ends and the Born nodes. These compose over the grid, carry the Born weights across each grid point by Lemma 26 and its one-sided limits at the nodes, and, by the Markov property of the process at the grid points, give each event of the grid values the weight $\rho_0(x_0)\prod_j T_j(x_j, x_{j+1})$. With records whose values no later stage couples, these T_j meet the hypotheses of (i) to (iii), and the Born weight of a record string at the end of its last write is that of its record-projected state. □

A.7 Independence in quantum equilibrium

Proof of Lemma 30. (i) A product state evolved by a sum of one-sided terms stays a product. In quantum equilibrium the actual variables on the slice where the settings are produced are distributed by its Born weight, the product of the weights of the two sides, so the two sides are independent there, and a setting kernel reading the generator side alone then makes the joint distribution of λ and (a, b) the product of its marginals. (ii) For each setting pair the correlation under $\rho(\cdot \mid a, b)$ differs from that under ρ by at most $2\delta_{ab}$, since the product of the two outcome expectations lies in $[-1, 1]$, and under ρ the CHSH bound $|S| \le 2$ holds [18], the absorbed local configurations being independent given λ. □

A.8 The apparent horizon from local data

Work in ingoing Vaidya form $ds^2 = -(1-2M(v)/r)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$, with $M \in C^1$, $M > 0$, $M' \ge 0$, and $M \to m_f < \infty$ as $v \to +\infty$. Write $f = 1 - 2M(v)/r$.

Apparent horizon. $\mathcal{N}_v = \{v = \text{const}\}$ is an ingoing null hypersurface. On $\mathcal{N}_v$ the outgoing radial null direction is $\ell = \partial_v + \frac{1}{2} f\,\partial_r$ and the ingoing one $n = -\partial_r$, with $\ell \cdot n = -1$. The expansion of a round sphere $S(v, r)$ along ℓ is, up to a positive normalization,

$$\theta_+(v, r) = \frac{f}{r} = \frac{r - 2M(v)}{r^2}. \tag{1}$$

At positive radius $\theta_+ = 0$ has the unique root $r = 2M(v)$, the round marginally outer trapped surface of $\mathcal{N}_v$, and the union of these spheres is the trapping horizon of [40]. Equation (1) contains $M(v)$ and no $M'(v)$, so two mass functions agreeing at v carry the same apparent horizon there.

Event horizon. The event horizon $\partial J^-(\mathcal{I}^+)$ is a null hypersurface $r = R(v)$ with outgoing null generators,

$$R'(v) = \tfrac{1}{2}\Big(1 - \frac{2M(v)}{R}\Big). \tag{2}$$

Under frozen M the fixed point $R = 2M$ has linearization $1/(4M) > 0$ and repels forward in v, so the physical generator is the solution selected by the future condition $R(v) - 2M(v) \to 0$ as $v \to +\infty$, a functional of $M|_{[v,\infty)}$.

The excess. Write $E(v) := R(v) - 2M(v)$. Since $1 - 2M/R = E/R$, (2) gives the linear equation $E' = E/(2R) - 2M'$. With the integrating factor $\mu(v) = \exp\big(-\int^v \frac{dv'}{2R(v')}\big) > 0$, $(\mu E)' = -2\mu M'$, and the future condition gives

$$E(v) = \int_v^\infty K(u, v)\, M'(u)\, du, \qquad K(u, v) = 2\exp\Big(-\int_v^u \tfrac{dv'}{2R(v')}\Big) > 0.$$

Hence $E(v) \ge 0$, $E(v) > 0$ on any interval where $M' > 0$ to the future of v, and $E \equiv 0$ exactly when M is constant to the future of v. Two non-decreasing mass functions can agree at every $v' \le v$ and settle to distinct final masses, so no map whose argument is the data on and before $\mathcal{N}_v$ returns $R(v)$, while the apparent-horizon radius is such a map. □

A.9 The homogeneous driver

Proof of Proposition 20. With zero shift, $t' = N$ is the definition of the lapse, and the homogeneous slice at proper time t has mean curvature $K = -3H(t)$, so the driver is $N' = -N^2(K - K_\star) = 3N^2(H(t) - H(\tau))$. Let $\Phi(x, y) = -3\int_y^x (H(s) - H(y))\, ds$ and $W(N) = 1/N - 1 + \log N$. Then $\Phi \geq 0$, because H is nonincreasing, and $W \geq 0$ for $N > 0$, with $W(N) \to \infty$ as $N \to 0$. Along a solution with $N > 0$,

$$\frac{d}{d\tau}\big(\Phi(t, \tau) + W(N)\big) = -3\big(H(t) - H(\tau) - H'(\tau)(t - \tau)\big) \leq 0,$$

because H' is nondecreasing. So $W(N) \leq V_0 := \Phi(t_0, \tau_0) + W(N_0)$ wherever the solution exists. The lapse therefore does not reach zero, $\log N \leq V_0 + 1$, and, since $\log x \leq x/2$ for $x > 0$, $1/N \leq 2(V_0 + 1)$:

$$\frac{1}{2(V_0 + 1)} \leq N \leq e^{V_0 + 1}.$$

With $t' = N > 0$ this gives $t \geq t_0$, and on a bounded parameter interval t and N stay in a compact set. On the open set $t > t_0/2$, $\tau > \tau_0/2$ the right-hand side is continuous and locally Lipschitz in (t, N), since $|H'(t)| \leq |H'(t_0/2)|$ there. The solution therefore exists and is unique locally, extends to a maximal interval, and, staying in a compact set on every bounded interval, extends to every $\tau \geq \tau_0$ [61, Thms. 2.2, 2.13, Cor. 2.16]. Finally $t' = N \geq 1/(2(V_0 + 1))$, so t is strictly increasing and unbounded, and by continuity it takes every value in $[t_0, \infty)$. □

A.10 The equivariant jump laws at a present

Proof of Proposition 33. (i) The master equation at P gives $\dot{P}_i = \sum_j (F_{ij} - F_{ji})$. Let $C = F - F^{\mathsf{T}} - J$; it is antisymmetric, since Hermiticity makes J antisymmetric, and equivariance, $\sum_j (F_{ij} - F_{ji}) = \sum_j J_{ij}$ for every i, is the vanishing of its row sums. For real x and y, $x = \max(0, x - y) + \min(x, y)$; with $x = F_{ij}$ and $y = F_{ji}$ this gives $F = \max(0, J + C) + S$ with $S = \min(F, F^{\mathsf{T}})$, symmetric and nonnegative. Conversely, for $F = \max(0, J + C) + S$ with C antisymmetric and S symmetric, $F - F^{\mathsf{T}} = \max(0, J + C) - \max(0, -(J + C)) = J + C$ and $\min(F, F^{\mathsf{T}}) = S$, so C and S are fixed by F and J, and F is equivariant exactly when the row sums of C vanish. At positive Born weights the rates F_{ca}/P_a are nonnegative and have flow F. The jump law has flow $\max(0, J)$; the further rate adds the symmetric flow $\lambda_{ac}|\psi_a||\psi_c|$, and the minimum of $\max(0, J_{ij})$ and $\max(0, J_{ji})$ is zero. (ii) Where $H_{ij} = H_{ji} = 0$, $J_{ij} = 0$, so $F_{ij} = \max(0, C_{ij}) + S_{ij}$ and $F_{ji} = \max(0, -C_{ij}) + S_{ij}$, and both vanish exactly when $C_{ij} = 0$ and $S_{ij} = 0$, since $S_{ij} \geq 0$. (iii) A flow vanishing off the edges of G is a function on its $2e$ directed edges, and its divergence at a configuration is the outflow minus the inflow. By (i) the net flow of an equivariant flow is J plus a cycle term, whose row sums vanish, so two equivariant flows have the same divergence and differ by a divergence-free flow. The divergence map to $\mathbb{R}^n$ has as image the vectors whose sum vanishes on each component, of dimension $n - k$, so its kernel has dimension $2e - n + k$. A symmetric flow is divergence-free, which gives a subspace of dimension e; the antisymmetric flows are the functions on the e oriented edges, and the same count on them gives $e - n + k$. At a member F_0 positive on every edge of G and zero off them, $F_0 + \epsilon X$ is nonnegative for small $\epsilon > 0$, vanishes off G, and has the divergence of F_0, so it is the flow of an equivariant law with rates $(F_0 + \epsilon X)_{ca}/P_a$. The complete graph has $e = n(n - 1)/2$ and $k = 1$, which gives $(n - 1)^2$. For the three-configuration example, the current $2\,\mathrm{Im}(\bar{\psi}_i H_{ij} \psi_j)$ vanishes at the uniform amplitude, each configuration receives and sends the flow $\kappa/3$, and $C = F - F^{\mathsf{T}}$ has the entry $\kappa/3$ from each configuration to the next. □

A.11 Law-dependent custody

Proof of Proposition 34. Write $\rho_x = |\psi_x|^2$, J_{xy} for the current into x from y, and $A = J + C$. Each ρ_x is a finite sum of terms $e^{-i(E - E')t}$, so the node times, where some amplitude not

identically zero vanishes, are isolated.

(i) Let the distinction be the proposition that the configuration at an earlier slice lay in M, with a truth value at every complete present of positive weight from t^* on. For $t > t^*$ and x of positive weight at t, the configurations at that slice admitted by the present lie on one side of M; call that side the label of x at t. *Between nodes.* On an interval beyond t^* free of node times every configuration of positive weight stays so, and the rates are continuous. The bound on staying gives $P_{\sigma,t}(x,x) > 0$ for $\sigma < t$, so the admitted earlier configurations at σ remain admitted at t and the label of x is constant on the interval. If flow passed at σ between configurations of different labels, the bound on staying, jumping, and staying would give a later present admitting earlier configurations on both sides of M. So the flows in both directions vanish across the labels; since their sum is $|A_{xy}| + 2S_{xy}$ and $S \geq 0$, $A_{xy} = 0$ and $S_{xy} = 0$ on every pair of different labels. *At a node.* Let $t_0 > t^*$ be a node time, and let D be the set of configurations, with amplitude not identically zero, whose labels on the two intervals adjacent to t_0 differ. A configuration of positive weight at t_0 keeps positive weight and continuous rates near t_0, so staying keeps its label across t_0: every $x \in D$ has $\rho_x(t_0) = 0$. Suppose D nonempty, let $W = \sum_{x\in D} \rho_x$, and let p be the least vanishing order at t_0 of the amplitudes of D. By the Leibniz rule, $W^{(n)}(t_0) = 0$ for $n < 2p$ and $W^{(2p)}(t_0) = \binom{2p}{p} \sum_{x\in D} |\psi_x^{(p)}(t_0)|^2 > 0$. Equivariance gives $W' = \sum_{x\in D,\, y\notin D} A_{xy}$, since J is antisymmetric and C is antisymmetric with zero row sums. For $x \in D$ and $y \notin D$ the label of y is the same on both sides and that of x is not, so the pair has different labels on the left interval or on the right one. Let f_- be the sum of A_{xy} over the pairs of the first kind and f_+ over the rest; f_- vanishes on the left interval and f_+ on the right one, and both are $2p-1$ times continuously differentiable near t_0, since p is at most the greatest vanishing order there. A C^k function vanishing on one side of t_0 has its derivatives through order k zero at t_0, so $W^{(2p)}(t_0) = f_-^{(2p-1)}(t_0) + f_+^{(2p-1)}(t_0) = 0$, a contradiction. So the labels agree across every node beyond t^*, the partition is the same at every later present, and by the first step no flow crosses it. By equivariance the weight of each label class then changes by the flux of A across its boundary, which vanishes, and a finite sum of terms $e^{-i(E-E')t}$ constant on an interval is constant. *The order bound.* With $E_1, \ldots, E_r$ the distinct energies of the state, $\psi_x(t) = \sum_m c_{xm} e^{-iE_m t}$, and $\psi_x^{(n)}(t_0) = \sum_m c_{xm} e^{-iE_m t_0} (-iE_m)^n$. If these vanish for $n < r$, the Vandermonde matrix of the distinct $-iE_m$ forces every $c_{xm} = 0$. So a node has order at most $r-1$.

(ii) With $\Omega = a + g$ and $\theta = \Omega t$, $\psi = (\cos\theta, -i\sin\theta, \cos\theta, -i\sin\theta)/\sqrt{2}$ solves the Schrödinger equation, by substitution. Put $k = \frac{1}{2}\sin 2\theta$. The currents into 1 from 0 and into 3 from 2 are ak, those into 1 from 2 and into 3 from 0 are gk, and the node times are those of $\theta \in \frac{\pi}{2}\mathbb{Z}$. On the state $c = g\max(0,k) + a\max(0,-k)$; c is continuous, and each c_i vanishes when an amplitude of its pairs does, so no flow leaves an empty configuration. $C = cZ$ is antisymmetric with zero row sums and lives on coupled pairs. The flow $\max(0, J + cZ)$ is Ωk from 0 to 1 and from 2 to 3 where $k \geq 0$, and $\Omega|k|$ from 1 to 2 and from 3 to 0 where $k \leq 0$, and zero otherwise; its net flow is J. On each half period between nodes two configurations drain, each into one successor and with no inflow, and the other two fill with no outflow. For a draining x with successor $x+1$, $P_{u,t}(x,x) = \rho_x(t)/\rho_x(u)$ and $P_{u,t}(x,x+1) = 1 - \rho_x(t)/\rho_x(u)$, and $P_{u,t}(y,y) = 1$ for a filling y; these solve the forward equation, are stochastic, compose, and carry the Born weights, since $\rho_x + \rho_{x+1}$ is constant on the half period. Over a half period each draining configuration therefore reaches its successor with probability one. During the n-th half period the histories from configuration 0 at parameter 0 occupy $\{n, n+1\}$ modulo 4 and those from 2 the other two, so every complete present of positive weight admits a single configuration at parameter 0 and the proposition is true or false there.

Under the jump law, on $0 < \theta < \pi/2$ the flows are ak and gk from 0 into 1 and into 3, and gk and ak from 2 into 1 and into 3, while 1 and 3 send nothing. By the window bound of Appendix A.4, histories from 0 and from 2 both jump to 1 with positive probability on a window inside $(0, \pi/2)$ and then stay, so at $\theta = \pi/2$, where only 1 and 3 have weight, each of

them admits both 0 and 2. Every later present of positive weight is reached from 1 or 3 with positive probability, so it admits both, and the proposition has no truth value there. Under the law with the further rate the rates add terms $\lambda_{xy}|\psi_y|/|\psi_x|$; at 1 and 3 these stay bounded up to $\theta = \pi/2$, where $\rho_1 = \rho_3 = \frac{1}{2}$, so staying there keeps positive probability and the same conclusion holds. The parameter $\theta = \pi/2$ is $\pi/(2(a+g))$. □

A.12 Recorded readouts

Proof of Proposition 35. Let U_A and U_B^θ solve $i\dot{U} = H_A U$ and $i\dot{U} = H_B^\theta U$ with initial value the identity; for Hermitian terms piecewise continuous in the parameter they exist, are unique, and are unitary. Then $V = U_A \otimes U_B^\theta$ satisfies $i\dot{V} = H_A U_A \otimes U_B^\theta + U_A \otimes H_B^\theta U_B^\theta = (H_A \otimes I + I \otimes H_B^\theta)V$ with $V(0) = I$, so by uniqueness it is the propagator of H, even where same-wing terms at different parameters do not commute. The amplitude at the present is $\psi^\theta = (I \otimes U_B^\theta)\varphi$ with $\varphi = (U_A \otimes I)\psi$, independent of θ. For a set M of local configurations the Born weight is $\sum_{a\in M}\sum_b |\psi^\theta(a,b)|^2 = \sum_{a\in M}\|U_B^\theta \varphi(a,\cdot)\|^2 = \sum_{a\in M}\|\varphi(a,\cdot)\|^2$, the same under both settings. By the equilibrium hypothesis this is the weight of the paths whose local configuration at the present lies in M, so every local readout at the present has the same law under both settings. On every path of positive weight the readings taken at earlier parameters equal a function m of the local configuration at the present, so the paths with readings y have the weight of the paths whose local configuration at the present lies in $m^{-1}(y)$, which is the Born weight of that local set. □

A.13 Setting dependence of the path law

Proof of Proposition 36. Let ν be a common component, $m = \sum_\omega \nu(\omega)$, and $r_{ab} = \mu_{ab} - \nu \geq 0$, of mass $1 - m$. Then $E_{ab} = \sum_\omega \nu A_a B_b + \sum_\omega r_{ab} A_a B_b$, and the second sum has absolute value at most $1 - m$. The first sums contribute $\sum_\omega \nu(\omega)\big(A_0(B_0 + B_1) + A_1(B_0 - B_1)\big)(\omega)$ to S, and pointwise $|A_0(B_0+B_1)+A_1(B_0-B_1)| \leq |B_0+B_1|+|B_0-B_1| \leq 2$, so with $\nu \geq 0$ they contribute at most $2m$ in absolute value. So $|S| \leq 2m+4(1-m)$, that is $1-m \geq (|S|-2)/2$. The pointwise minimum of the four laws is nonnegative and below each, so it is a common component, and every common component lies below it. □

Proof of Corollary 37. The pointwise minimum of $p(z)\mu_{ab}(\omega \mid z)$ over the four pairs is $p(z)$ times the minimum of the conditional laws, so the noncommon residual mass of the greatest common component is $\sum_z p(z)\big(1 - \sum_\omega \min_{ab} \mu_{ab}(\omega \mid z)\big)$. By Proposition 36 applied to the conditional laws, each bracket is at least $(|S_z| - 2)/2$, and it is nonnegative, so it is at least $\frac{1}{2}(|S_z| - 2)_+$. Every common component has at least this residual mass. Since $S = \sum_z p(z) S_z$, $|S| - 2 \leq \sum_z p(z)(|S_z| - 2) \leq \sum_z p(z)(|S_z| - 2)_+$. □

A.14 The dynamics in two minisuperspace sectors

Proof of Proposition 38. Reduction. For either metric with homogeneous scalars, direct computation of the Einstein tensor shows that the Euler-Lagrange equations of $\sqrt{-g}\,(R/(2\kappa) - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i)$ in (N, a, b, ϕ_i) are the diagonal Einstein-scalar equations, and the off-diagonal ones hold identically. In the variables q the gravitational kinetic term is a negative multiple of $(ab^2/N)(\dot{s}^2 - \dot{\alpha}^2)$, so the Hamiltonian is $\tilde{N}C$ with C as stated; the curvature of the two-spheres gives $V = -c\,e^{2s}$, and the flat slices of $\mathbb{T}^3$ give $V = 0$. Here ℓ is the coordinate volume of a slice, and $p_i = \ell\, ab^2\, d\phi_i/dt$ in the proper time t. A term $\tilde{N}U$ added to $\tilde{N}C$, for a function U of q, is the reduction of a homogeneous source of energy density $\rho_U = U/(\ell a^2 b^4)$ and principal stresses $-\rho_U - a\,\partial_a\rho_U$ and $-\rho_U - \frac{b}{2}\,\partial_b\rho_U$, and adds the force $-\tilde{N}\partial_{\phi_i}U$ to the scalar equations.

State. For $f = u + iv$, $-\operatorname{Im}(\bar{f}f') = vu' - uv'$, the Wronskian of two solutions of a linear equation with no term in f', so w_k is constant and nonzero. Each term $e^{-iE\alpha}e^{ik\phi_1}f_k(s)$ satisfies

$\hat{C}\Psi = (\frac{1}{4c}(E^2 f_k + f_k'') + \frac{k^2}{2\ell} f_k + V f_k) e^{-iE\alpha} e^{ik\phi_1} = 0$. For the superposition,

$$-\operatorname{Im}(\bar{\Psi}\partial_s \Psi) = w_0 + \varepsilon^2 w_1 + \varepsilon X, \qquad X = -\operatorname{Im}(e^{i\phi_1} \bar{f}_0 f_1' + e^{-i\phi_1} \bar{f}_1 f_0'),$$

and X is bounded on the compact window, so $j > 0$ for $|\varepsilon| < w_0/\sup|X|$. On $\mathbb{T}^3$, $w_k = \nu_k$ and $X = (\nu_0 + \nu_1)\cos(\phi_1 - (\nu_1 - \nu_0)s)$, so $-\operatorname{Im}(\bar{\Psi}\partial_s\Psi) \geq (\nu_0 - \varepsilon\nu_1)(1-\varepsilon) > 0$ at every s for $0 \leq \varepsilon < \nu_0/\nu_1 < 1$. Where $j > 0$, $R > 0$, $\nabla S = \operatorname{Im}(\nabla\Psi/\Psi)$ is smooth, $j = R^2(-p_s)/(2c)$ gives $p_s < 0$, and $\tilde{N}$ is positive and finite. Ψ, and hence R, ∇S, and Q, depend on ϕ_1 and s only, $\partial_\alpha S = -E$, and $\partial_{\phi_2} S = 0$.

(i). With $\Psi = Re^{iS}$ and G constant,

$$\frac{\hat{C}\Psi}{\Psi} = C(q, \nabla S) + Q - \frac{i}{2R^2}\,\partial_i(R^2 G^{ij}\partial_j S),$$

so $\hat{C}\Psi = 0$ gives $C(q,\nabla S) + Q = 0$ and $\partial_i(R^2 G^{ij}\partial_j S) = 0$ at every configuration where $j > 0$. A relation holding at every such configuration holds at every point of every path and at every destination of a jump. A jump moves neither α nor s, keeps p_s by construction, and $p_\alpha = -E$ everywhere.

(ii). Between jumps, for $g \in \{\alpha, s\}$, $\dot{g} = \tilde{N}\,\partial C/\partial p_g$ at $p = \nabla S$, and $\frac{d}{d\lambda}\partial_g S = \sum_j \dot{q}^j \partial_j \partial_g S = \tilde{N}\sum_j (\partial C/\partial p_j)\,\partial_g\partial_j S$. Differentiating $C(q, \nabla S(q)) + Q(q) = 0$ with respect to g gives $\sum_j (\partial C/\partial p_j)\,\partial_g\partial_j S = -\partial_g(C+Q)$ at fixed momenta. So $\dot{p}_g = -\tilde{N}\partial_g(C+Q)$, and the term from the configuration dependence of $\tilde{N}$ carries the factor $C + Q = 0$. The geometry and its momenta therefore obey the Hamilton equations of $\tilde{N}(C+Q)$, with the scalar momenta $\nabla_\phi S$ and Q read at the actual configuration; by the reduction with $U = Q$ these are the Einstein equations of the sector with the complete source, the stress of energy density and pressures $\sum_i p_i^2/(2\ell^2 a^2 b^4)$ at $p_i = \partial_{\phi_i} S$ together with the source of $U = Q$, and between jumps the contracted Bianchi identity conserves that source. By (i) the energy of the complete source equals $-(p_\alpha^2 - p_s^2)/(4c) - V(s)$, a function of s, p_α, and p_s, which no jump changes, so it is continuous on every path. On every path p_s is continuous, and $\dot{p}_s = -\tilde{N}(V'(s) + \partial_s Q)$ is bounded and smooth between jumps. So $\dot{\alpha} = \tilde{N} G^{\alpha\alpha} p_\alpha = E/p_s$, $\dot{s} = 1$, and $N = ab^2\tilde{N}$ are continuous, with bounded derivatives that are smooth between jumps, and with $dt = N\,d\lambda$ the functions a and b are continuously differentiable in t with bounded second derivatives, smooth between jumps. The energy density ρ is fixed by the first derivatives of the metric through the Hamiltonian constraint, so it is continuous, and the energy equation $\dot{\rho} + \frac{\dot{a}}{a}(\rho + P_1) + 2\frac{\dot{b}}{b}(\rho + P_2) = 0$, in which the principal stresses P_1, P_2 enter undifferentiated and bounded, holds across the jumps in integrated form.

(iii). $\dot{s} = \tilde{N} G^{ss} p_s = 1$. Between jumps $d\phi/ds = \tilde{N}\nabla_\phi S/\ell = J/j$ with $J = R^2\nabla_\phi S/\ell$, and, since $\partial_\alpha(R^2 G^{\alpha\alpha}\partial_\alpha S) = 0$, the continuity equation of (i) reads $\partial_s j + \nabla_\phi \cdot J = 0$ on the torus. The flow of the smooth field J/j therefore carries the density $j(s_a, \cdot)$ to $j(s_b, \cdot)$, and $\int j\,d\phi$ does not depend on s. At a jump on the slice s, j does not depend on ϕ_2, so a uniform ϕ_2 keeps the density. The function $h = p_s(s, \cdot)$ is real analytic on the circle; where it is constant only ϕ_2 is redrawn, and otherwise it has finitely many critical points, and on each interval of monotonicity the substitution $c = h(\phi_1)$ gives $\int F\,j\,d\phi_1 = \int dc \sum_{h(\phi_1')=c} F\,j/|h'|$ at ϕ_1'. Drawing ϕ_1 from the solutions of $h = c$ with weights $j/|h'|$ is therefore drawing from the density j conditioned on the value of h, which keeps the density. The jump times form a Poisson process independent of the configuration, so conditional on them every flow segment and every jump keeps the density $j/\int j$ on its slice.

(iv). Every jump redraws ϕ_2 uniformly. Uniformly on the window, $\partial_s\Psi/\Psi = f_0'/f_0 + \varepsilon e^{i\phi_1}(f_1/f_0)' + O(\varepsilon^2)$, so the first Fourier coefficient of $p_s = \operatorname{Im}(\partial_s\Psi/\Psi)$ in ϕ_1 is $\frac{\varepsilon}{2i}(f_1/f_0)' + O(\varepsilon^2)$. If $(f_1/f_0)'$ vanished on an interval, then $f_1 = \mu f_0$ there and the two equations would give $(2c/\ell)\mu f_0 = 0$, while $\mu \neq 0$ and $f_0 \neq 0$ because $w_1 \neq 0$ and $w_0 \neq 0$. So at some s the coefficient is nonzero for every sufficiently small $\varepsilon \neq 0$.

On $\mathbb{T}^3$ let $\delta = \nu_1 - \nu_0$, $D = 1 + 2\varepsilon\cos\theta + \varepsilon^2$, and $A' = \varepsilon(\varepsilon + \cos\theta)/D$. Then $\Psi = e^{-iE\alpha - i\nu_0 s}(1 + \varepsilon e^{i\theta})$, $R^2 = D$, and $S = -E\alpha - \nu_0 s + \arctan\big(\varepsilon\sin\theta/(1+\varepsilon\cos\theta)\big)$, whose derivative

in θ is A', so $p_s = -\nu_0 - \delta A'$ and $\partial_{\phi_1} S = A'$. As a function of $\cos\theta$, A' has derivative $\varepsilon(1-\varepsilon^2)/D^2 > 0$, so the solutions of $p_s(s,\phi_1') = p_s(s,\phi_1)$ are θ and $-\theta$, that is ϕ_1 and $2\delta s - \phi_1$; j and $|\partial_{\phi_1} p_s| = \delta\varepsilon(1-\varepsilon^2)|\sin\theta|/D^2$ take equal values at the two, so each has probability $\frac{1}{2}$. With $2c/\ell = \nu_1^2 - \nu_0^2$ the guidance gives $d\theta/ds = \tilde{N}\partial_{\phi_1} S/\ell - \delta = \delta\nu_0(2A'-1)/(\nu_0 + \delta A')$, and

$$\frac{dp_s}{ds} = -\delta\,\frac{dA'}{d\theta}\,\frac{d\theta}{ds} = \frac{\delta^2\nu_0\,\varepsilon(1-\varepsilon^2)\sin\theta\,(2A'-1)}{D^2(\nu_0+\delta A')},$$

which takes opposite values at θ and $-\theta$, nonzero when $\sin\theta \neq 0$ because $A' < \frac{1}{2}$ and $\nu_0 + \delta A' > 0$. The two continuations have equal p_s at the jump and opposite nonzero dp_s/ds, so up to the next jump p_s differs between them at every s of an interval after the jump, with one sign there. On that interval $d\alpha/ds = E/p_s$ and $N = 2c\,e^s/(-p_s)$ differ, so α and the lapse differ.

(v). The slices are orbits of the symmetry group, with compact connected leaves $S^1 \times S^2$ or $\mathbb{T}^3$ and Riemannian induced metric; the group acts on each by isometries that preserve its unit normal, so its mean curvature is constant. Along every path, a and b are continuous and positive and $N = ab^2\tilde{N} > 0$, so on every compact range of s the coordinate speed of a causal curve is bounded by $N/\min(a,b)$ and λ increases strictly along it. An inextendible causal curve therefore meets every slice of the window exactly once, so each slice is a Cauchy surface of the realized geometry over the window.

On $\mathbb{T}^3$ the bound of *State* holds at every s, $\tilde{N}$ is bounded, and the flow exists for all s. For each n the process started at $s = -n$ with density $j/\int j$ has that density on every later slice by (iii), so by the Markov property the processes for different n agree where both are defined and extend to one process on all s. The argument of the previous paragraph, applied on every compact range of s, makes each slice a Cauchy surface, so the realized geometry is globally hyperbolic. Since $A' \geq -\varepsilon/(1-\varepsilon)$, $-p_s \geq m = \nu_0 - \delta\varepsilon/(1-\varepsilon)$, and the added condition gives $m > \nu_0/2 = E/2$. The proper time $\int N\,ds = \int 2c\,e^s/(-p_s)\,ds$ is finite toward $s \to -\infty$. With dots now derivatives in the proper time t, the Kretschmann scalar of $-dt^2 + a^2dx^2 + b^2(dy^2+dz^2)$ is $4\big[(\ddot{a}/a)^2 + 2(\ddot{b}/b)^2 + 2(\dot{a}\dot{b}/ab)^2 + (\dot{b}/b)^4\big]$, and $\dot{b}/b = (E - p_s)e^{-s}/(6c) \geq Ee^{-s}/(6c)$, so it diverges. With $\ln a = (2\alpha + s)/3$, $\ln b = (s-\alpha)/3$, and $d\alpha/ds = E/p_s \in [-E/m, 0)$, the integrands of $\int dt/a$ and $\int dt/b$ in s are $(2c/(-p_s))e^{(2s-2\alpha)/3}$ and $(2c/(-p_s))e^{(2s+\alpha)/3}$; α increases toward $s \to -\infty$ and $d(2s+\alpha)/ds \geq 2 - E/m > 0$, so both converge, and the comoving coordinates of every past-inextendible causal curve converge. Beyond (v), the following is argued rather than proved in full: the future of such a curve is the future of the comoving worldline through its limit point, so the past causal boundary is the image of the connected torus under a map continuous in the chronological topology. It is connected, and the realized geometry is one domain [Definition 2]. □

A.15 Refinement of the present fiber

Proof of Lemma 45. For an inserted parameter u between s and t,

$$P_{s,t}(a,c) = \sum_b P_{s,u}(a,b)P_{u,t}(b,c)$$

with nonnegative terms, so $P_{s,t}(a,c) > 0$ exactly when some b has $P_{s,u}(a,b) > 0$ and $P_{u,t}(b,c) > 0$. The histories over the refined parameters, restricted to the original ones, are therefore exactly the histories over the original parameters, and a proposition about the presents at the original parameters holds on all, none, or some of the one set exactly when it does on the other. Induction on the number of inserted parameters completes the proof of the first statement. For a slice parameter r earlier than τ_0 the Born weight on S_{τ_0} is the image of the Born weight on S_r, $\rho_{\tau_0}(b) = \sum_a \rho_r(a)P_{r,\tau_0}(a,b)$ with nonnegative terms, so $\rho_{\tau_0}(b) > 0$ exactly when some a has $\rho_r(a) > 0$ and $P_{r,\tau_0}(a,b) > 0$. The histories from S_r, restricted to the original parameters, are therefore exactly the histories from S_{τ_0}, and a proposition about the presents at the original parameters holds on all, none, or some of the one set exactly when it does on the other. □

A.16 The fiber as slice traces

Proof of Proposition 48. Write $\rho_x = |\psi_x|^2$ and let $v_0 < \cdots < v_M$ be the grid, through $\tau_0, \ldots, \tau_N$. For an analytic Hamiltonian each Born weight is real analytic on the interval [Appendix A.3], so one not identically zero has finitely many zeros on $[\tau_0, \tau_N]$, and the grid through the slice parameters and these zeros has the dichotomy.

The process. On an open grid interval the rates out of a configuration of positive weight are nonnegative and continuous, since H and ψ are continuous and the weight does not vanish there, and every rate into a configuration whose amplitude vanishes there is zero, since $J_{ij} = 0$ where $\psi_i = 0$. The flow $\sum_{i \neq j} \max(0, J_{ij})$ is at most $C \sum_x \rho_x$ for C at least every off-diagonal row sum of $|H_{ij}|$, so the expected number of jumps over a grid interval is finite and a path has finitely many jumps there with probability one. Joined at the grid points by the Markov property, this gives one process on $[\tau_0, \tau_N]$. From Born-distributed data its one-time law is the Born weight at every parameter [Lemma 26], and its transition probabilities between grid points compose and carry the Born weights, with values into and out of a grid point the one-sided limits. They reach from configurations of positive weight only configurations of positive weight.

Positivity on a grid interval. Let a have positive weight at the start s of a grid interval and c at its end u. Both then have positive weight throughout (s, u), by the dichotomy and continuity. The transition probability from a at s to c at u is positive exactly when some path from a to c has finitely many jumps at strictly increasing parameters in (s, u), each at positive rate, through configurations of positive weight. Let such a path exist. A configuration of positive weight throughout a closed subinterval stays over it with positive probability, since its exit rate is bounded there. A jump at positive rate at a parameter where the rate is continuous occurs near that parameter with positive probability. Near s the row from a tends to the unit row, and near u, where the weight of c is bounded away from zero, c stays up to u with positive probability. The Markov property chains these into a positive transition probability. Conversely, the process enters no configuration whose amplitude vanishes on the interval and jumps only at positive rate, so a positive transition probability is carried by paths of that kind.

The grid. A sequence of configurations at the grid points whose first has positive weight and whose every step has positive transition probability has, on each grid interval, a path of that kind; joined, they form a path of the process with finitely many jumps, each at positive rate, of positive weight at every parameter of $[\tau_0, \tau_N]$, whose values at the grid points are the sequence. Conversely the values at the grid points of such a path have a first member of positive weight and, interval by interval, positive transition probabilities.

The slice parameters. The transition probability between consecutive slice parameters is the composition of those between the grid points from one to the other, so it is positive exactly when a chain of positive steps through those grid points joins its ends, as in the proof of Lemma 45, and the values of a path at the slice parameters are the restriction of its values at the grid points. The two sets therefore agree at the slice parameters. The state evolves by its own law, so a sequence of complete presents is fixed by its sequence of configurations.

The further rate. For analytic H the further rate is nonnegative, continuous out of configurations of positive weight, and zero into configurations whose amplitude vanishes; its flow is symmetric, so the Born weights still solve the master equation [Appendix A.3], and its flow adds at most $L \sum_x \rho_x$ for L at least every row sum of λ. The argument applies unchanged. □