Branch Indexical Theorem: A No-Go for Observable Single-History Selection

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Abstract

A single-history selection law is branch indexical when it marks one branch by an inaccessible label while leaving the unitary dynamics unchanged. The Branch Indexical Theorem identifies a quiescent class of such laws and shows every member is operationally silent on the post-decoherence pointer algebra, a conditional no-go for observable selection within that class. Silence is membership in an equivalence class induced by restriction to that algebra, so the strength of the claim is fixed by the choice of algebra.

The boundary carries the contribution. One-way diagnostics on measurable functionals detect a population shift, generated inter-block coherence, an affinity defect, and the failure of local realizability, each implying a named restriction fails; no converse holds, and a null reading certifies nothing. Mechanistic amplitude blindness admits no output-level diagnostic. And leaving the class does not purchase observability: a state-dependent law can exit quiescence and remain silent, so restriction violation is necessary for empirical content and does not secure it.

Three results carry the classification beyond exact idealizations, bounding the deviation under finite calibration error and finite decoherence, and closing the multishot channel through history-wise conditional fidelity.

The result does not derive the Born rule, solve the measurement problem, choose an ontology, or introduce a new empirical prediction; Born universality is derived from calibration on a spanning family with affinity rather than assumed. The framework separates branch selection, branch actuality, and the truthmaker of actuality, and names which commitments a putatively empirical single-history proposal has already made.

Contents

1 Introduction **3**

2 The Quiescent Class **7**

3 The Branch Indexical Theorem **12**

4 Operational Diagnostics at the Quiescence Boundary **17**

5 Silent Ontology or Paid Exit **23**

6 No-Signaling Consistency **24**

7 Escape Routes **26**

8 Falsifiability: Lindblad Flow Exit and Experimental Bounds **28**

9 Single-History Selection in the Quiescent Class as Silent Ontological Structure 30

10 Discussion 31

A Proof Kernel 34

B Formal Verification 34

C Limits of Operational Discrimination 37

D Experimental bounds landscape 38

How to read this paper

BIT addresses two audiences: foundations of QM specialists who want the operational no-go, and philosophy of physics readers who want the metaphysical accounting of single-history selection. The technical core is load-bearing for both audiences; the paths below cut the paper into reading recipes.

The claim. The conditional no-go is stated in the abstract and proved in Theorem 11. The four-restriction class is Definition 5; the fork between silent and paid is Corollary 30. A reader who wants only the claim shape can stop there.

Minimal reading path. The full claim without the witness machinery:

- Abstract
- Definition 5 (quiescent class)
- Theorem 11 (BIT)
- Corollary 30 (silent ontology or paid exit)
- Section 10

Technical validation path. The proof kernel, the boundary witnesses, no-signaling for the state-dependent escape, and the operator-level verification probes:

- Definition 5 (quiescent class) with the channel domain paragraph for R_4
- Theorem 11 (BIT) plus Appendix A (proof kernel)
- Theorem 25 (operational diagnostics), including the R_1 dichotomy lemma (Lemma 35)
- Proposition 33 (no-signaling for the state-dependent escape)
- Appendix B (formal verification: F1 kernel, F9 bridge, T7 generalized no-signaling, C8 energy neutrality)

Metaphysics path. The typed Actual/Selected/Truthmaker apparatus and the truthmaker gap payoff:

- Key terms (end of Section 1)
- Theorem 11 (BIT)
- Remark 20 (Everettian comparison as discrimination protocol surrogate)
- Remark 23 (operational restatements)
- Section 9 (silent ontological structure and the truthmaker gap payoff)

Conventions and notation. Selected(k, e) is the operational primitive at the BIT layer; Actual(k, e) is the framework-realized actuality predicate; Truthmaker(Actual(k, e)) is what the realizing framework must supply. The full glossary is in the key-terms block at the end of Section 1. Throughout, "Observable" means a self-adjoint element of $\mathcal{A}_P$, the post-decoherence pointer algebra; claims about "operational silence" are scoped to $\mathcal{A}_P$.

1 Introduction

The Branch Indexical Theorem (BIT; Theorem 11) classifies single-history selection laws by their operational footprint on the post-decoherence pointer algebra. The result is realization-neutral: it applies to any framework that supplies Truthmaker(Actual(k, e)) for the selected branch at each commitment event e. Candidate realizers include presentism in the sense of Prior-Bigelow [5, 25], Kent's real-world single-history quantum mechanics [22–24], and the growing-block view; BIT does not certify that any of them supplies the truthmaker, since presentness is slice-indexed while Actual is branch-label-indexed and no coercion between the two indexings is given here. The Kent comparison is the one argued in this paper. BIT does not resolve which framework supplies the truthmaker, nor does it engage Putnam-Rietdijk-style challenges to a preferred slice; those are constraints on the realizing framework and lie outside the scope of this paper. The natural question for foundations of quantum mechanics is whether such a framework-supplied actuality fact can carry a pointer-algebra operational signature when paired with the unitary formalism. BIT answers the question for the natural class of candidates and finds them silent.

The unactualized outcome at a commitment event. Table 1 classifies the ontological status of the unactualized outcome at a commitment event; the taxonomy is a scope clarifier for Definition 5, which operates on Case 2 structure.

Table 1: Three-case taxonomy of the unactualized outcome at a commitment event. BIT (Definition 5) operates on Case 2 structure.

Case	**Example**	**Status of the unactualized**
Classical Contingency	Dice roll	Same-world possibility: nomologically possible in the same world; did not occur this run.
Decoherence / branch selection	Pointer decoherence (Sample operator)	Decoherent records for non-committed branches; their ontological status (retained but inaccessible, objectively collapsed, or counterfactual) is supplied by the realizing framework.
Quantum Indeterminism	Atom decay	Counterfactual: no other branch produced; the unactualized has no corresponding physical structure.

The result. BIT's scope is post-decoherence single-history selection (Case 2 of Table 1: decoherent branch selection at a commitment event, distinct from classical contingency and quantum indeterminism). BIT identifies a zero signature regime: within the quiescent class (Definition 5), a single-history selection ontology reproducing unitary quantum mechanics and Born weights contributes nothing to any expectation taken in the post-decoherence pointer algebra. The selection fact is present in the ontology and absent from the accessible statistics. This equivalence result is distinct from Proposition 33: the former proves a population map identity on diagonal pointer inputs, while the latter proves invariance of the remote marginal under local selection, for the safe subclass of Lemma 35.

Why the result is constraining. BIT is not a positive prediction. Its purchase is foreclosure: every rival single-history selection law that reproduces standard QM and hopes to carry an empirical signature must either fail at least one quiescence restriction on Sample or relax the exactly unitary Flow premise. Each such law then faces a bound whose class depends on the exit channel. The energy injection family (GRW, CSL, Diosi-Penrose, adding non-unitary localization dynamics) is constrained by spontaneous-emission and CMB channels [13, 30]; a nonlinear amplitude-reading exit (dropping state-independence) is constrained by Gisin-Polchinski no-signaling [17, 27], and the remaining exit classes face their own channels. Section 7 maps representative exits to their associated empirical channels and constraint families. A state-dependent selection that reads the realized amplitudes exits quiescence but stays silent by preserving pointer-covariance; no-signaling is secured by Proposition 33. The four restrictions play distinct roles in the proof of Theorem 11: R_3 and R_4 force the identity step at the run-ensemble level, R_2 blocks coherence creation, and R_1 is classificatory (separating amplitude-reading from non-reading selection); see Remark 10.

Proof structure. The diagnostic theorem (Section 4) carries the substantive work: for each restriction it exhibits a channel that relaxes it and opens a representative empirical channel on a stated witness domain, together with the further restrictions that must be relaxed alongside it. Only R_3 admits a single-restriction witness; the remaining three are relaxed in named sets, which is itself a structural fact about the class and no gap in the constructions. The contribution is therefore the boundary map.

What this paper does. This paper proves BIT, establishes the load-bearing witness channels at the quiescent class boundary, gives two operational restatements of the equivalence, and maps the paid exits. It is self-contained on the theorem, the equivalence, the escape-route map, and the silent ontology classification, and it classifies the operational status of single-history selection laws.

Key terms and notation.

BIT Branch Indexical Theorem (Theorem 11).

SelLaw The typed single-history selection law $\mathcal{S}$ acting at each commitment event; subject to the four quiescence restrictions R_1–R_4. Distinct from the per run selector SelFact that implements it.

SelFact The per run amplitude-blind selector implementing SelLaw; typed $\Lambda \to \{1, \ldots, n\}$ (mechanistic (α)-typing of R_1).

k^* The inaccessible label produced by SelFact at each commitment event e (i.e., $k^* = k^*(e)$). k^* is not a function of any element of $\mathcal{A}_P$; it is the operational primitive on which the indexical-selection concept rests.

Selected(k, e) Operational primitive at the BIT layer: the proposition that label k is the one produced by SelFact at commitment event e, i.e., $k = k^*(e)$. Selected is a typed bookkeeping predicate over $\Lambda \times$ Events; it carries no ontological weight beyond the operational record.

Actual(k, e) Framework-realized actuality predicate: a typed placeholder for the realizing framework's truthmaker on the selection. BIT does not identify Actual with Selected, with operational accessibility, with pointer occupation, with Born weight, with observer-relative record formation, or with membership in the Everettian comparison surrogate. Whether the truthmaker is monadic presentness, relational selection, final-boundary selection, growing-block inclusion, or haecceistic thisness is left to the realizing framework.

Truthmaker(Actual(k, e)) The realizing framework's burden: what makes Actual(k, e) true beyond the operational Selected(k, e). Deferred to the realizing framework; BIT proves silence over the operational layer and does not supply this truthmaker.

Branch indexical (selection) A single-history selection mechanism that picks one branch by reference to the inaccessible label k^*: the operational predicate is Selected(k, e), asserting that the branch indexed by $k = k^*(e)$ is the one produced by SelFact at event e. Whether Selected lifts to Actual in the realizing-framework sense is the truthmaker burden, and lies outside BIT. The selection leaves the unitary dynamics unmodified and couples k^* to no element of $\mathcal{A}_P$.

Observable In this paper, a self-adjoint element of $\mathcal{A}_P$. All "observable"/"observability" claims are scoped to $\mathcal{A}_P$; quantities outside the pointer algebra are not addressed by BIT.

Operationally silent (or inert) A selection mechanism whose action on the post-decoherence pointer marginal is the identity on $\mathcal{A}_P$: no observable shift, no energy injection through Hamiltonians in the pointer algebra, no pointer-coherence change, no probability shift, no dynamical (Flow) modification.

Quiescent class Single-history selection laws satisfying R_1–R_4; the operationally silent class (taxonomy of unactualized-outcome status in Table 1; BIT operates on Case 2 structure).

R_1 State-independence: the per run selector SelFact has no p-port (it does not read realized amplitudes).

R_2 Pointer-covariance: the selection preserves the post-decoherence pointer basis and creates no new pointer coherence.

R_3 Born calibration: under one fixed rule the run-ensemble reproduces $p_k = |a_k|^2$ on an affinely spanning family of amplitude assignments. Universality across the simplex is not assumed; it follows with R_4 (Proposition 6).

R_4 Linear CPTP on the pointer marginal: the run-ensemble averaged map is a linear completely-positive trace-preserving channel.

(α) Typing constraint on SelFact (mechanistic sense of R_1): the per run selector SelFact : $\Lambda \to \{1, \ldots, n\}$ has no p-port; amplitude-blindness is a fact about the type, and no side constraint on a kernel value.

(β) Ensemble-level silence (operational sense of R_1): the induced map $M = \mathbb{E}_\lambda[\text{Sample}]$ acts as the dephasing channel $\mathcal{D}_A^\theta$ on the pointer marginal. $(\alpha) \Rightarrow (\beta)$ under Born-faithfulness and linear CPTP; the converse fails on the broader class of p-indexed kernels (Remark 10(2); construction in Appendix C).

(γ) Definitional restatement of the carrier: Sample propagates Born weights already present in the post-decoherence population $p_k = \text{tr}(P_k \sigma)$ instead of generating them.

Sample The commitment operator that fires at each decoherence event; selects one branch as the realized history.

Flow The unitary evolution layer between Sample firings; standard quantum dynamics.

Operational diagnostics The tightness result exhibiting, for each R_k, a channel that relaxes it and the set relaxed alongside it (single-restriction for R_3 only; sense (2) uniquely diagnostic for R_1, R_3 only; see Theorem 25).

$\mathcal{C}_1, \ldots, \mathcal{C}_4$ The four representative witness channels, one per R_k.

$\mathcal{A}_P$ The post-decoherence pointer algebra (abelian von Neumann algebra generated by pointer projectors $\{P_k\}$). That is to say, the observables that only ask which decohered pointer branch happened, without seeing anything inside or between branches.

M The induced population map: the action of Sample on the pointer simplex.

$\mathcal{D}_A^\theta$ The dephasing channel on Alice's marginal at pointer basis setting θ.

Born-faithful A selection law whose per-outcome conditional weight equals the Born weight $p_k = |a_k|^2$ (the postulated probability for branch k). Equivalently, the run-ensemble action of the selection equals the pointer dephasing channel on the post-decoherence marginal. Formal definition at Definition 8.

Gisin-Polchinski no-signaling The theorem that nonlinear quantum evolution between entangled subsystems would enable superluminal signaling, hence is forbidden by relativistic causality [17, 27]. In BIT, this rules out amplitude-reading exits outside the Born-faithful pointer-covariant safe subclass (Lemma 35, Proposition 33).

Helstrom bound $P_{\text{disc}}^{\text{opt}} = \frac{1}{2} + \frac{1}{2} D(\rho, \sigma)$ with $D(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1$ [18]; saturation at 1/2 is operational indistinguishability.

Lindblad Flow exit Relaxation of exactly unitary Flow by a single dissipator in the Markovian CPTP generator $d\rho/dt = -i[H, \rho] + \sum_k (L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\})$, keeping R_4 intact at the integrated-channel level; BIT's primary falsifiable channel (Section 8).

Markovian Memoryless: evolution depends on the present state alone, with no integral over prior times. Retrocausal and two-state-vector accounts violate Markovianity via future boundary data (Section 7).

Popperian Relating to Karl Popper's falsifiability criterion: a claim is scientific in Popper's sense if some possible empirical observation could refute it.

Scope and nonclaims. BIT is a targeted no-go on a defined class. The following lie outside its claim:

- BIT does not derive the Born rule (the postulate that outcome probabilities equal $|a_k|^2$, the squared amplitudes); R_3 postulates Born calibration on an affinely spanning family, and universality across the simplex is derived from calibration together with R_4 (see Remark 19).
- BIT does not solve the measurement problem; the per run truthmaker for Actual(k, e) is exactly the measurement-problem burden carried by the realizing framework (Section 9, "Truthmaker gap"), and BIT conditions on such a framework instead of discharging the burden.

- BIT does not select a foliation, that is, a preferred slicing of spacetime into simultaneity surfaces fixing which slices count as "now" (Remark 15).
- BIT does not prove maximality over all non-quiescent laws; the diagnostic theorem delivers boundary witnesses only (see the per-witness scope statements in Section 4).
- BIT does not classify unconstrained temporal or process level correlations, which remain open; the subclass satisfying history-wise conditional fidelity is closed by Proposition 14 (see Remark 32).

BIT stays at the operational level and leaves foliation, truthmaker, and metaphysical-reality questions to the realizing framework. It delivers pointer-algebra silence for the quiescent class on a single round, together with representative boundary witnesses for each R_k exit.

Roadmap.

- Section 3 states and proves BIT; Remark 23 gives two operational restatements.
- Section 4 establishes the load-bearing witness channels, and Section 6 secures no-signaling for the state-dependent escape.
- Section 7 consolidates rival programs by exit class. Section 9 classifies quiescent single-history selection as silent ontological structure, and Section 10 summarizes what BIT establishes.
- Appendix A carries the proof kernel; Appendix B carries the formal verification.

2 The Quiescent Class

This section defines the quiescent class through four restrictions on Sample: R_1 types the per run selector as amplitude-blind, and R_2 through R_4 constrain the run-ensemble map. The proof of the equivalence case of Theorem 11 consumes R_2, R_3, and R_4; R_1 types the class without entering that proof (Lemma 35).

Definition 1 (Pointer algebra and post-decoherence population carrier)**.** The *pointer algebra* $\mathcal{A}_P$ is the abelian von Neumann algebra generated by the post-decoherence pointer projectors $\{P_k\}_{k=1}^n$, of ranks $r_k = \operatorname{tr} P_k$. The post-decoherence pointer marginal is the Born-weighted population

$$\sigma \;=\; \sum_k p_k\,\tau_k, \qquad \operatorname{supp}\tau_k \subseteq \operatorname{ran} P_k, \quad \operatorname{tr}\tau_k = 1, \qquad p_k \;=\; \operatorname{tr}(P_k\,\sigma) \;=\; |a_k|^2,$$

with population vector $p \in \Delta_n$ (the standard n-simplex). Each τ_k is the normalized state within block k; the populations are read as $\operatorname{tr}(P_k\sigma)$ throughout, never off a diagonal entry. Where every P_k is rank-1 the blocks are one-dimensional, $\tau_k = P_k$, and the marginal collapses to $\sigma = \sum_k p_k P_k$; that case is named explicitly wherever it strengthens a conclusion, and is not assumed here. Nothing in the silence results depends on the within-block states τ_k; no element of $\mathcal{A}_P$ separates them. p is a datum supplied by state preparation and decoherence; it is fixed before Sample acts and is not generated by the selector. The *branch registry* T_p is the post-decoherence branch registry equipped with the branch measure p (measure-valued, and not a branch-counting or token-multiplicity structure).

Definition 2 (Operational equivalence induced by $\mathcal{A}_P$)**.** For states ρ, ς on the pointer Hilbert space, write

$$\rho \sim_{\mathcal{A}_P} \varsigma \quad \text{iff} \quad \operatorname{tr}(O\rho) = \operatorname{tr}(O\varsigma) \;\text{ for every } O \in \mathcal{A}_P.$$

This is an equivalence relation, and by Definition 1 its classes are the level sets of the population vector: $\rho \sim_{\mathcal{A}_P} \varsigma$ exactly when $\mathrm{tr}(P_k\rho) = \mathrm{tr}(P_k\varsigma)$ for every k. Two states in one class differ, if at all, in inter-block coherence and in their within-block states τ_k, neither of which any element of $\mathcal{A}_P$ resolves. Where every P_k is rank-1 and both states are block-diagonal, the class is a single state and $\sim_{\mathcal{A}_P}$ is equality.

The relation is what the silence results are stated in. A selection law is operationally silent when it maps each admissible input into its own class, and the strength of every such claim is fixed by the choice of $\mathcal{A}_P$: enlarging the algebra refines the classes and weakens the claim, restricting it does the reverse.

Definition 3 (Restricted trace distance and the population diagnostics)**.** The metric on the classes of Definition 2 is the trace distance restricted to $\mathcal{A}_P$,

$$D_{\mathcal{A}_P}(\rho,\varsigma) \;=\; \tfrac{1}{2}\sum_k \big|\mathrm{tr}\big(P_k(\rho-\varsigma)\big)\big|,$$

the classical trace distance between the two population vectors. It is bounded by 1, it vanishes exactly on $\sim_{\mathcal{A}_P}$-classes, and it is the optimal discrimination advantage available to a measurement in $\mathcal{A}_P$: an observer restricted to that algebra succeeds with probability $\frac{1}{2} + \frac{1}{2}D_{\mathcal{A}_P}$. Every population claim below is stated in it.

Three further functionals measure the ways a selection law can depart from quiescence. For a run-ensemble map Φ and an input ρ,

$$\Delta_p(\Phi,\rho) \;=\; D_{\mathcal{A}_P}\big(\Phi(\rho),\rho\big), \qquad \Gamma(\Phi,\rho) \;=\; \sum_{j\neq k}\big\|P_j\,\Phi(\rho)\,P_k\big\|_1,$$

$$\Delta_{\mathrm{aff}}(\Phi;\rho,\varsigma,x) \;=\; \big\|\Phi\big(x\rho+(1-x)\varsigma\big) - x\Phi(\rho) - (1-x)\Phi(\varsigma)\big\|_1,$$

the population shift, the inter-block coherence generated, and the affinity defect. In the bipartite setting Δ_B denotes the trace distance between Bob's marginals at two settings of Alice. All four are nonnegative, vanish on a quiescent law, and are what Theorem 25 reads.

A χ^2 rate is a finite-sample statistic and is used only as such. It is not a metric on states: it is normalization-dependent, and it diverges as a population approaches zero where $D_{\mathcal{A}_P}$ stays bounded. Where a χ^2 rate appears below it accompanies the corresponding $D_{\mathcal{A}_P}$ or Γ statement and never replaces it.

Definition 4 (SelFact typing (R_1))**.** For each post-decoherence amplitude vector $p \in \Delta_n$, the per run selector is the map

$$\mathrm{SelFact}_p : \Lambda \times T_p \to T_p,$$

where $(\Lambda, \mathcal{F}, \mu)$ is a probability space indexing per run randomness and T_p is the branch registry of Definition 1 (a measured space carrying the Born measure p). The displayed signature $\mathrm{SelFact} : \Lambda \to \{1,\dots,n\}$ used in body prose is the label projection of $\mathrm{SelFact}_p$; the subscript is implicit elsewhere.

Restriction R_1 (state-independence, mechanistic (α)-typing). The selector does not compute, inspect, or condition on the coordinate vector p; p-dependence enters only through the input measure class of T_p. Equivalently, $\mathrm{SelFact}_p$ has no p-port: there is no measurable dependence of the selector on the realized amplitude vector p, and amplitude-blindness is a type fact, not a side constraint on a p-indexed kernel. The selector reads labels of already-instantiated branches; the identity $\mu \circ \mathrm{SelFact}_p(\cdot)^{-1} = p$ holds by the branch measure on T_p, and by no property of μ or Λ.

Induced population map. The run-ensemble pushforward $M = \mathbb{E}_\lambda[\mathrm{SelFact}_p] : \Delta_n \to \Delta_n$ acts on the population carrier of Definition 1.

Born weights live in the carrier; Price of R_1. The quiescent selector is not a mechanism for generating Born frequencies; it is a mechanism for marking one element of an already Born-measured decohered branch registry. The Born content lives in the carrier (state preparation and decoherence supply p); SelFact reads labels from T_p and never amplitudes from the state. The factoring is deliberate: R_3 declares Born universality as an input constraint on the carrier, and the (α)-typing of SelFact declares amplitude-blindness as a constraint on the selector. The typing is what closes the Gisin-Polchinski amplitude-reading channel here; no further argument about the carrier is required.

Price of R_1. R_1 does not explain Born weighting. It assumes a branch registry already equipped with Born measure. Therefore BIT excludes amplitude-reading selection mechanisms but does not explain why the registry has Born measure. The result is a no-go conditional on Born-measured branch availability.

Definition 5 (Quiescent class)**.** A single-history selection law on the carrier of Definition 1 is *quiescent* if it satisfies the following restrictions. The first constrains the per run selector; the rest constrain the run-ensemble map Φ.

(i) R_1 *carrier-relative amplitude blindness:* the selector satisfies the SelFact typing of Definition 4. It does not inspect state coordinates; all Born dependence is delegated to the measured carrier. This is not state-independence in the stochastic-kernel sense, and Proposition 7 states what it is instead.

(ii) R_2 *pointer-covariance,* in two strengths that the results below keep apart. The *weak* form R_2^w is diagonal preservation, $\mathcal{D}\Phi\mathcal{D} = \Phi\mathcal{D}$: a block-diagonal input has a block-diagonal image, so no inter-block coherence is created on the decohered carrier. The *covariant* form R_2 is $\mathcal{D}\Phi = \Phi\mathcal{D}$ on all inputs. Covariance implies diagonal preservation, since $\mathcal{D}^2 = \mathcal{D}$; the converse fails, because a map may leave block-diagonal inputs alone while reading inter-block coherence into output populations.

(iii) R_3 *Born calibration:* under one fixed rule the law reproduces $p_k = |a_k|^2$ on an affinely spanning family of amplitude assignments; the n pointer eigenstates suffice. Born weights on *every* assignment are not assumed here. They follow, by Proposition 6, from calibration together with R_4.

(iv) R_4 *affinity:* the run-ensemble map on the pointer marginal is affine under probabilistic mixtures and completely positive and trace-preserving. Affinity is a condition on the map's response to preparing mixtures, not a bookkeeping convenience: it is what carries calibration off the spanning family.

For the bipartite constructions a further condition is named separately, because it concerns the extension of the law to a joint system and not the selector or its pointer marginal.

- R_L *local realizability:* the nonselective action on a joint state is realized by a local trace-preserving map, $\Lambda_A \otimes I_B$, on Alice's factor alone.

Proposition 6 (Born universality from calibration and affinity)**.** *Under R_3 and R_4 the induced population map satisfies $M(p) = p$ for every $p \in \Delta_n$. Certainty preservation on the pointer eigenstates, together with affinity under probabilistic mixtures, forces the Born population map on the entire simplex.*

Proof. Write $p = \sum_i p_i e_i$ over the vertices of Δ_n. Calibration gives $M(e_i) = e_i$ for each i, and affinity gives $M(p) = \sum_i p_i M(e_i) = \sum_i p_i e_i = p$. □

Proposition 7 (Carrier–selector trilemma)**.** *A fixed probability space (Λ, μ), a fixed p-independent measurable selector $f : \Lambda \to \{1, \ldots, n\}$, and exact Born frequencies for arbitrary $p \in \Delta_n$ cannot all obtain for $n \geq 2$. Born dependence must enter through the selector, through the measure on Λ, or through the carrier measure.*

Proof. A fixed f and fixed μ determine the single distribution $\mu \circ f^{-1} \in \Delta_n$, which cannot equal an arbitrary p for $n \geq 2$. □

The trilemma fixes what R_1 claims. This paper takes the third horn: the carrier T_p supplies the Born measure and the selector reads labels from it. Currying p into the type of the selector's input does not remove the dependence, and R_1 is not the claim that it does. It is the claim that the dependence sits in the carrier rather than in the selector's mechanism, which is what makes the amplitude-reading channel of Lemma 35 closed at the typing level.

Channel domain for R_4. R_4 is a statement about the nonselective, run-ensemble map

$$\Phi : \mathcal{T}(\mathcal{H}_P) \to \mathcal{T}(\mathcal{H}_P)$$

on the trace-class operators of the pointer Hilbert space, or, in the strictly post-decoherence reduction, about the induced classical stochastic map

$$M : \Delta_n \to \Delta_n$$

on the pointer simplex. When "CPTP" is used in R_4, Φ is the operator-level channel with Choi matrix $J_\Phi \geq 0$ and $\mathrm{tr}_{\mathrm{out}}\, J_\Phi = I_{\mathrm{in}}$ (Choi positivity for complete positivity and partial-trace identity for trace preservation). When only M is used, complete positivity is inherited from the block embedding $\Delta_n \hookrightarrow \mathcal{T}(\mathcal{H}_P)$, $p \mapsto \sum_k p_k \tau_k$ at a fixed choice of within-block states $\{\tau_k\}$: a row-stochastic map on Δ_n lifts to a CPTP map on the block-diagonal subspace, and the two formulations agree on the post-decoherence marginal. The lift depends on the chosen $\{\tau_k\}$ and the agreement is on $\mathcal{A}_P$, which is where every claim below is stated.

The measurement problem. The per run implementation of SelFact (which k^* is realized in a particular run) is the measurement problem and is not fixed by the typing of Definition 4. BIT's theorem level claim is the ensemble level statement. The carrier selector split of Definitions 1 and 4 is a structural device carrying no ontological commitment: it pre-commits to no Bohmian, modal-actualist, or other metaphysical reading of SelFact.

The question of whether Sample operates as a branch-selection (decoherent records for non-committed branches; Case 2) or as an indeterministic actualization (no other branch produced; Case 3) is the measurement problem; BIT classifies the operational consequences of single-history selection laws without solving it.

Definition 8 (Born-faithful selection)**.** A single-event selection law S_θ is *Born-faithful* if, in the orthogonal record limit ($\langle E_j \mid E_k \rangle = \delta_{jk}$ on the environment record states), it outputs pointer branch k with probability $p_k = \mathrm{tr}(P_k\, \sigma)$ for every post-decoherence pointer state σ, and assigns the corresponding conditional reduced state $|k\rangle\langle k|_A$ to the pointer register on that branch. Equivalently, the nonselective run-ensemble action of S_θ agrees with the pointer dephasing channel on the remote marginal:

$$\mathrm{tr}_A\, \mathbb{E}[S_\theta(\sigma)] = \mathrm{tr}_A\, \mathcal{D}^\theta_A(\sigma), \qquad \mathcal{D}^\theta_A(\sigma) := \sum_k (P^\theta_k \otimes I_B)\, \sigma\, (P^\theta_k \otimes I_B),$$

where $P^\theta_k = |k_\theta\rangle\langle k_\theta|_A$.

Remark 9 (Which clause the proofs use). Clause (ii) is the operative condition used in the proof of Proposition 33, and it is stated at the marginal level because that is the level at which it holds for every post-decoherence σ. The corresponding full-state identity $\mathbb{E}[S_\theta(\sigma)] = \mathcal{D}^\theta_A(\sigma)$ is strictly stronger and holds only in the orthogonal record limit; a pointer-diagonal unitary is the standing counterexample outside it (Proposition 33, Step 2). Clause (i) is the physical interpretation in the pointer-decoherence regime, where the orthogonal record condition holds as a regime feature; clauses (i) and (ii) coincide on that regime, and (ii) is the form used in proofs. The output-probability clause follows from the dephasing identity by taking the A-diagonal.

Operator pipeline (notation). Throughout the paper, the commitment process is factored into two layers acting on records that decoherence has already produced:

- Flow: unitary evolution between commitments.
- Sample: the single-history selection rule.

The quiescence restrictions are properties of Sample and of the run-ensemble average $\mathbb{E}$[Sample] on the pointer marginal. These labels serve only to make the restrictions on Sample and $\mathbb{E}$[Sample] precise.

Remark 10 (Joint uniqueness of the identity realizer). The two distinctions below separate state-independence and Born universality and remove the appearance of tension between them.

1. **Joint forcing of the identity.** The combination of Born universality (iii) and linearity (iv) forces the run-ensemble population map $\mathbb{E}_\lambda$[Sample] to identity (established in Theorem 11); R_1 classifies the mechanism as amplitude-nonreading. An amplitude-blind sampler would produce Born statistics only on the measure-zero equiprobable class ($\mathcal{C}_3$ of Theorem 25 is the counterexample), which is why R_3 enters as a separate restriction. Sample is no independent sampler but a bookkeeping selection propagating Born weights already present in the ensemble. Quiescence constrains the admissible form of Sample; silence follows only once (iii) and (iv) force the induced population map to the identity.

2. **One-way implication of** (α) **and** (β)**.** The typing constraint on SelFact (sense (α)) implies the ensemble level condition $M = \mathbb{E}_\lambda[\text{Sample}] = \mathcal{D}_A^\theta$ (sense (β)) under Born-faithfulness (Definition 8, clause (ii)) and linear CPTP (R_4); the converse does not hold for the broader class of p-indexed kernels that lie outside the SelFact type.

   Forward direction $(\alpha) \Rightarrow (\beta)$. State-independence forces $M = \mathbb{E}_\lambda$[Sample] to act on the pointer diagonal without reading p at any individual run; Born-faithfulness then forces $M = \mathcal{D}_A^\theta$ on every diagonal input, since the only state-independent linear map from probability vectors to probability vectors that fixes every Born-weight vector is the identity on the simplex (the dephasing channel on diagonal inputs).

   Reverse direction $(\beta) \not\Rightarrow (\alpha)$*: counterexample.* A per run kernel can read the realized amplitudes (violating (α)) while its run-ensemble average still equals $\mathcal{D}_A^\theta$ (satisfying (β)); the explicit $\sin(2\pi\lambda)$-modulated p-indexed kernel construction and validity check is given in Appendix C. That kernel inhabits the broader class of p-indexed stochastic kernels (those with a p-port), which lies outside the SelFact type of Definition 5(i); the SelFact typing is strictly stronger than the ensemble level silence condition.

   Correct relationship. (α) is the mechanistic (per run) non-reading condition and (β) the operational (ensemble level) silence condition. The per run measurement problem (which run is selected) sits at the (α) level, and (β) is silent on it. BIT's theorem level claim (Theorem 11) uses (β) directly; when R_1 is imposed at the (α) level (as in Definition 5(i)), (β) follows from the forward direction above, and the converse is nowhere needed in the proof.

   Definitional status of (γ). Under the carrier selector split of Definition 5(i), (γ) is a definitional restatement of the carrier: Sample propagates Born weights already present in the population carrier without generating them. A reader who prefers "genuine metaphysical selector whose per run effects cancel in aggregate" is describing the same typed object and is not excluded by the content of R_1; the typing fixes amplitude-blindness, not the metaphysics of SelFact.

3 The Branch Indexical Theorem

This section establishes the Branch Indexical Theorem on the post-decoherence pointer algebra fixed in Section 2; clause (a) gives operational equivalence with the Everettian comparison construct of Remark 20, and clause (b) gives the silence of the induced pointer marginal map that drives the equivalence.

Theorem 11 (Branch Indexical Theorem)**.** *Let a single-history selection model reproduce the dynamics of unitary quantum mechanics exactly on the unitary evolution layer* Flow *between commitment events (*Sample *firings; see Section 2) and weight outcomes by the Born rule. Then:*

(a) ***Statistics within the quiescent class.*** *For any selection law in the quiescent class (Definition 5), the augmented model is $\sim_{\mathcal{A}_P}$-equivalent to the unaugmented one (Definition 2): the statistics predicted on the post-decoherence pointer algebra are those of the unitary formalism, and the selection fact contributes nothing to any expectation taken in $\mathcal{A}_P$. Conditional on a branch, the predicted statistics are identical to standard quantum mechanics.*

(b) ***Foreclosure of observable indexical selection.*** *A quiescent selection fact is observationally silent: its action on the post-decoherence pointer marginal is the identity (coherence-neutral, and energy-neutral for Hamiltonians in the pointer algebra), and the Helstrom bound (the optimal success probability for distinguishing two quantum states by measurements in the stated algebra) on distinguishing the selection applied from the no selection state is saturated as a theorem level equality at $1/2$, not approached as a sensitivity-limited bound; the discriminating measurements range over $\mathcal{A}_P$, and over all measurements when the pointer projectors are rank-1. Any selection structure that hopes to carry a pointer-algebra operational signature under the post-decoherence single-round scope must drop at least one of the four restrictions and exit quiescence.*

Proposition 12 (Finite calibration error)**.** *Suppose the induced population map is affine (R_4) and calibrated on the vertices only to within $\|Me_i - e_i\|_1 \le \delta_i$, and set $\delta_{\max} = \max_i \delta_i$. Then for every $p \in \Delta_n$,*

$$\|Mp - p\|_1 \;\le\; \sum_i p_i \delta_i \;\le\; \delta_{\max},$$

and the discrimination advantage available in $\mathcal{A}_P$ obeys $D_{\mathcal{A}_P}(\Phi(\sigma), \sigma) \le \frac{1}{2}\delta_{\max}$.

Proof. By affinity, $Mp - p = \sum_i p_i (Me_i - e_i)$; the triangle inequality and $\sum_i p_i = 1$ give the bound. □

Exact calibration is therefore not required for the conclusion, only bounded calibration error, and the bound is uniform over the simplex. This is the independent reason R_3 is stated on a spanning family: the vertices are what an experiment can calibrate, and affinity is what propagates a vertex-level tolerance to every input.

Proposition 13 (Robustness under approximate decoherence)**.** *Let Φ be completely positive and trace-preserving and leave every block-diagonal state $\mathcal{A}_P$-silent, $\Phi(\varsigma) \sim_{\mathcal{A}_P} \varsigma$ whenever $\varsigma = \mathcal{D}\varsigma$ (the conclusion of Theorem 11(b)). If the input satisfies $\|\rho - \mathcal{D}\rho\|_1 \le \eta$, then*

$$D_{\mathcal{A}_P}(\Phi(\rho), \rho) \;\le\; \tfrac{1}{2}\eta.$$

Proof. Pointer dephasing preserves block weights, so $D_{\mathcal{A}_P}(\mathcal{D}\rho, \rho) = 0$, and the hypothesis gives $D_{\mathcal{A}_P}(\Phi(\mathcal{D}\rho), \mathcal{D}\rho) = 0$. The triangle inequality leaves $D_{\mathcal{A}_P}(\Phi(\rho), \Phi(\mathcal{D}\rho))$, which is at most $\frac{1}{2}\|\Phi(\rho) - \Phi(\mathcal{D}\rho)\|_1$ because the P_k are orthogonal; a trace-preserving positive map contracts the trace norm, so this is at most $\frac{1}{2}\|\rho - \mathcal{D}\rho\|_1 \le \frac{1}{2}\eta$. □

The carrier of Definition 1 is exactly block-diagonal, which no physical record is. The bound above is what connects the exact statement to a finitely decohered state: on the pointer algebra the deviation a quiescent law can produce is at most half the input's own departure from block diagonality, with no assumption that environmental records are perfectly orthogonal. The bound is stated on $\mathcal{A}_P$ because that is what Theorem 11(b) delivers at general pointer rank; at rank-1 the $\mathcal{A}_P$ statement is trace-norm closeness on the pointer register.

Proposition 14 (Sequential silence on the pointer history)**.** *Let $h_{t-1} = (k_1, \ldots, k_{t-1})$ be the realized pointer history before event t. Suppose that at every event the selection law reproduces the quantum conditional, $P_{\mathrm{Sel}}(k_t \mid h_{t-1}) = P_{\mathrm{QM}}(k_t \mid h_{t-1})$, and that the realized label enters* Flow *only through the standard conditional state. Then for every finite T,*

$$P_{\mathrm{Sel}}(k_1, \ldots, k_T) \;=\; P_{\mathrm{QM}}(k_1, \ldots, k_T),$$

so the law is silent on the finite pointer-history algebra and not merely on each single-shot marginal.

Proof. The chain rule factorizes each joint distribution into its conditionals, which agree termwise by hypothesis. □

Both hypotheses carry the result. Equality of unconditional one-shot marginals does not suffice: a selector with cross-run memory can match every marginal and still produce different joint statistics, which is the multishot channel left open at Remark 32. History-wise conditional fidelity is what closes it. A model seeking empirical content through temporal correlation alone must therefore pay in one of three places: altered conditionals, selector memory carried between events, or feedback of the realized label into Flow.

Remark 15 (Foliation neutrality)*.* BIT selects no foliation. The four restrictions are conditions on the induced pointer marginal map and on the run-ensemble population map, and none of them mentions a slicing; the operational statements are correspondingly ordering-independent for spacelike-separated commitment events, which is what the history-wise argument of Proposition 14 delivers. That proposition factorizes the joint pointer statistics through conditionals that agree with the quantum ones, and the quantum joint for spacelike-separated events does not depend on the order in which they are taken, so no reordering changes a prediction on $\mathcal{A}_P$. The residue is stated plainly: a commitment event selects one branch of a joint state across a spacelike pair, so a realizing framework that takes Selected(k, e) as a matter of fact supplies whatever slicing that selection needs. BIT neither supplies such a slicing nor forbids one, and the cost of supplying it is charged to the realizing framework (Remark 20).

Remark 16 (The Everettian comparison, and what it does not carry)*.* Clause (a) is often stated as a comparison: a quiescent single-history ontology is operationally indistinguishable, on $\mathcal{A}_P$, from Everettian unitary mechanics [15] augmented by an inaccessible label k^* marking the selected branch. The comparison is available and it is the vivid form of the result, but it establishes nothing. An augmentation by a label that is not a function of any element of $\mathcal{A}_P$ leaves every $\mathcal{A}_P$ expectation unchanged, by construction and without argument; the comparison is therefore trivial on the Everettian side. The content is on the single-history side, in clause (b), whose proof runs from R_2, R_3, and R_4 and never mentions a comparison object. The surrogate displays that result to a reader; it is no premise of it. Its scope is set in Remark 20: the label is a semantic slot for the discrimination protocol, not an Everettian beable, and it carries no truthmaker obligation on that side.

Status: classification, not new dynamics. Theorem 11 is a classification result: once Born universality and linearity are imposed on the post-decoherence pointer marginal, no residual selection degree of freedom remains visible in the pointer algebra. Its mathematical kernel

is one line of linear algebra (Appendix A), and the substantive work sits around that line. The kernel step is where R_3 and R_4 combine: calibration pins the population map on the vertices, which is where an experiment can fix it, and affinity carries that to the whole simplex (Proposition 6), tolerance included (Proposition 12). Clause (b) is proved from R_2, R_3, and R_4 alone, while R_1 types the class, which is why a state-dependent law can be silent (Lemma 35). The depth accordingly lives in the admissible-class definition (Definition 5), the diagnostic theorem (Theorem 25) with its one-way operational tests for departures from each restriction, and the escape-route map (Section 7) with its paid exits and constraint families. What the classification supplies is an accounting: which commitments a single-history proposal has already made once it claims to be empirically meaningful, and where ontology, selection, dynamics, and observability come apart. Each is separately typed here, and conflating any two of them is what makes an operationally silent structure look either empirically significant or ontologically idle.

Remark 17 (Structural silence and instrumental silence). Two claims of the form "no signature has been seen" are distinguished throughout, and the difference is not one of degree. Within the quiescent class the discrimination excess ε vanishes identically on $\mathcal{A}_P$, and it does so because k^* is not a function of any element of $\mathcal{A}_P$ (Definition 1, Remark 20): there is no port through which the selection fact could act on an accessible observable. In the vocabulary of Definition 2, k^* cannot separate $\sim_{\mathcal{A}_P}$-classes, and the zero follows from the typing and is no measurement outcome. A collapse-model bound has the opposite character. There ε is a positive function of model parameters, and the experimental claim is $0 < \varepsilon \ll$ the sensitivity of the apparatus.

The two are separated by what would refute them. A confirmed nonzero ε on $\mathcal{A}_P$ refutes structural silence outright, since the claim is an equality and admits no margin. The same observation, made against a sensitivity claim, is absorbed by revising the parameter downward. Structural silence is therefore the more exposed of the two, not the safer.

The exact zero is a property of the idealization, and on a carrier it is approached to a stated margin: a finitely decohered input bounds the $\mathcal{A}_P$ deviation by $\eta/2$ (Proposition 13), and a vertex calibration good only to $\delta_{\max}$ leaves $P_{\text{disc}} \leq 1/2 + \delta_{\max}/4$ (Proposition 12). Those margins do not turn the structural claim into a sensitivity claim, because neither is a parameter of the selection law. Each is fixed by the carrier and the calibration and shrinks with them, whereas a collapse-model ε is set by the model and survives every improvement in the apparatus.

Both regimes appear in this paper, so the contrast is checkable here. The quiescent class is silent by Theorem 11(b). The Lindblad Flow exit of Section 8 carries $\gamma > 0$ and is bounded by detector sensitivity in the ordinary way (Appendix D). The boundary between them is the fork of Corollary 30, and crossing it converts a structural zero into an instrumental bound.

The scope is the pointer algebra, and outside it the equality is not claimed: at higher pointer rank $\varepsilon = 0$ holds for discriminations in $\mathcal{A}_P$, and for all discriminations only when the pointer projectors are rank-1. A framework whose actuality fact is typed outside the accessible algebra incurs a permanent bound on its own empirical content there. That is the standing commitment of the position, stated once here and not revisited.

Remark 18 (Surviving escape hatches). The enumeration is scoped, but not exhaustive. The available exits from quiescence are: state-dependent selection (the law reads the realized amplitudes); non-CPTP or retrocausal accounts (Aharonov two-state-vector, transactional); nonlinear or contextual collapse [29]; environment-assisted signatures (the silence proof fixes only the reduced pointer state). The exits are consolidated in Section 7 together with the empirical channel each one triggers.

Remark 19 (Born non-derivability). BIT does not derive the Born rule; it isolates where Born universality must enter. An amplitude-blind selection law returns one fixed population vector whatever the amplitudes carried by the record, so it agrees with the Born weights at the single simplex point where that vector sits and nowhere else. For $n \geq 2$ that agreement set is null in the simplex, and amplitude-blindness and Born universality are therefore incompatible. Born

enters as an input constraint on the quiescent class and never as an output of Sample. The standard Everett obstruction reaches the same conclusion from the branch-counting side [21, 22].

Remark 20 (Operational surrogate scope of clause (a)). The dummy label k^* matches the single-history Selected-degree-of-freedom on the realizing-framework side (whose lift to Actual requires the framework's Truthmaker). The equivalence is operational (both sides are empirically inert on $\mathcal{A}_P$) and asserts no common ontological category for the two ontologies; the discrimination-protocol form is in Remark 23. Single-history-ness in BIT refers to the post-decoherence regime: at each commitment event e, one record is Selected(k, e) on the surviving timeline (and Actual(k, e) holds in the realizing-framework sense via its truthmaker). The status of the unselected records varies by interpretation (Everett retained but inaccessible, objectively collapsed, counterfactual); BIT takes no position within this range. On the single-history side, the operational silence of Selected(k^*, e) is the conclusion of clause (b) under R_1–R_4, reached through the identity population map, and is not a presupposition of inaccessibility. BIT is conditional on a realizing framework that supplies the truthmaker (presentism with its presentness predicate is one such framework; predicate-type metaphysics is outside the scope of this paper); BIT classifies only the operational footprint of that commitment.

Proof of clause (b); equivalence in clause (a) is its operational restatement. By the linear CPTP restriction (Definition 5(iv)), the population map M induced by any quiescent Sample map is linear; by Born universality (Definition 5(iii)), M fixes each element of a spanning subset of the pointer simplex. A linear map fixing a spanning set is the identity (the proof kernel, Appendix A; elementary linear algebra). Both restrictions are consumed here: without linearity the spanning set does not propagate, and without Born universality there is nothing fixed to propagate. The CPTP assumption supplies the physical admissibility of the averaged pointer marginal channel and rules out nonphysical ensemble maps; the spanning family is one of Born-weight vectors, not the realized population vector of a single run, which is what lets the kernel step propagate across the full pointer simplex. Energy neutrality follows for Hamiltonians in the pointer algebra, $H = \sum_k h_k P_k \in \mathcal{A}_P$, where the energy is a function of the block weights alone (Appendix B).
Bridge from population identity to $\mathcal{A}_P$-state silence. The chain has four steps; the restrictions each consumes are named as they enter.

(i) Input: $\sigma = \sum_k p_k \tau_k$ is block-diagonal by post-decoherence, with populations $p_k = \mathrm{tr}(P_k \sigma)$.

(ii) Pointer-covariance (R_2, read as $\mathcal{D} \circ \Phi = \Phi \circ \mathcal{D}$ where $\mathcal{D}$ is pointer-dephasing) gives $\mathcal{D}(\Phi(\sigma)) = \Phi(\mathcal{D}(\sigma)) = \Phi(\sigma)$: $\Phi(\sigma)$ is itself pointer-diagonal; no off-diagonal pointer coherence is created.

(iii) The population vector $\big(\mathrm{tr}(P_k\, \Phi(\sigma))\big)_k = M(p)$, and $M = I$ on Δ_n by $R_3 + R_4$ via the kernel argument (Appendix A, Appendix B); hence $\mathrm{tr}(P_k\, \Phi(\sigma)) = p_k$ for every k.

(iv) From (iii) alone: for every observable $O \in \mathcal{A}_P$,

$$\mathrm{tr}\big(O\Phi(\sigma)\big) \;=\; \mathrm{tr}\big(O\sigma\big).$$

$\mathcal{A}_P$ is generated by $\{P_k\}$, so every $O \in \mathcal{A}_P$ has the form $\sum_k o_k P_k$ and $\mathrm{tr}(O\rho) = \sum_k o_k \,\mathrm{tr}(P_k \rho)$ depends on ρ only through its block weights. Equality of those weights is therefore already equality on $\mathcal{A}_P$. Equivalently, $\Phi(\sigma) \sim_{\mathcal{A}_P} \sigma$: a quiescent selection maps each admissible input into its own equivalence class (Definition 2).

Step (iii) carries the $\mathcal{A}_P$ equality by itself: coherence created off the block diagonal is invisible to $\mathcal{A}_P$, so a map that moved no block weight but manufactured coherence would still pass step (iv). Step (ii) is what excludes that map, and it is required for clause (b)'s coherence-neutrality and for the rank-1 lift, not for the algebra equality. If all pointer projectors are rank-1, steps (ii) and (iii) together lift the $\mathcal{A}_P$-state equality to ordinary operator equality $\Phi(\sigma) = \sigma$ on the pointer register;

for rank-r blocks ($r > 1$) it remains equality of block weights, since $\mathcal{A}_P$ does not see intra-block content. The equality is at the $\mathcal{A}_P$-state level on the reduced pointer marginal, not merely agreement of one chosen measurement's frequencies (see Remark 23 for the formal trace-norm statement). The Helstrom bound on distinguishing the selection applied reduced state from the no selection reduced state at fixed post-decoherence pointer marginal saturates at 1/2 over measurements in $\mathcal{A}_P$, and over arbitrary measurements in the rank-1 pointer case, where $\mathcal{A}_P$-state equality is operator equality (Appendix B). The equivalence in clause (a) is the operational restatement: a silent selection produces statistics identical to no-selection (Everett); since k^* is inaccessible by construction (not a function of any observable in $\mathcal{A}_P$, per Remark 20), the EverettAug reduced state on the pointer algebra satisfies $\rho_{\text{EverettAug}}|_{\mathcal{A}_P} = \rho_{\text{Everett}}|_{\mathcal{A}_P}$; combined with Silence($S$) (which gives $\rho_{\text{select}}|_{\mathcal{A}_P} = \rho_{\text{Everett}}|_{\mathcal{A}_P}$), vanishing of the $\mathcal{A}_P$-restricted trace distance, and therefore clause (a), follow. The only difference between the two is a single-history selection fact that is not a function of any expectation value. □

Remark 21 (Energy neutrality requires the algebra, not commutation). The hypothesis $H \in \mathcal{A}_P$ is not interchangeable with $[H, P_k] = 0$. The commuting condition is weaker and does not suffice once a pointer block has rank above one: such an H may separate states inside a block, and the block weights do not determine its expectation. At rank-1 pointer projectors the two conditions coincide.

Remark 22 (Quiescent class scope and the state-dependent escape). Clause (a) is scoped: it equates the single-history and Everettian content *within* the quiescent class. A state-dependent selection (which reads the realized amplitudes) escapes the equivalence case through the dropped restriction (i); no-signaling for the state-dependent class is secured separately by Proposition 33, which shows that the run-ensemble average of a Born-faithful state-dependent selection is a dephasing channel on the relevant pointer marginal, saturating the Helstrom bound at 1/2 at the level of the reduced state. This escape route is available to any Born-faithful, pointer-covariant state-dependent selection; BIT applies to it via Remark 18 and Proposition 33, in place of clause (a). The R_1 dichotomy completing the picture (no-signaling Born-faithful state-dependent laws are absorbed into quiescence; signaling state-dependent laws are excluded by Proposition 33 and the Gisin-Polchinski bound) is Lemma 35.

What BIT establishes. Theorem 11 converts the standing observation that a single-history selection fact is operationally close to Everett-with-a-pointer from an interpretive coincidence into a structural result: within the quiescent class, the equivalence is exact at the post-decoherence pointer-algebra level, and the Helstrom bound on distinguishing the two is saturated as an equality at 1/2 over $\mathcal{A}_P$ measurements, and over all measurements when the pointer projectors are rank-1 (clause (b)). The result is therefore a no-go for observable single-history selection within the quiescent class, and no defeasible interpretive overlap. Any empirically distinguishable single-history selection law must exit through at least one of the four quiescence restrictions or relax exactly unitary Flow; the corresponding costs are catalogued in Section 7 and Section 8.

Remark 23 (Discrimination-protocol and operator-algebra restatements (definitional)). Theorem 11(a) admits two operational reformulations that make the silence claim legible without strengthening it.

Discrimination-protocol form. Consider two candidate completions of the same operational preparation with equal prior:

- H_p: a single-history ontology (within a realizing framework supplying the truthmaker) in which a single-history selection law $\mathcal{S} \in$ quiescent class is applied at each commitment, yielding a single Selected branch k^* (and Actual(k^*, e) via the framework's truthmaker) with Born-weighted probability;

- H_e: an Everettian unitary-mosaic history in which all branches persist with Born amplitudes and an inaccessible flag k^* is stipulated to mark the Selected branch. The flag k^* is a bookkeeping label carrying no truthmaker obligation on the Everettian side; it is a semantic slot introduced to match the single-history degree of freedom, not a thisness (haecceistic) fact about which branch is actual.

Bob discriminates between the candidate completions via a POVM $\{E_h, I - E_h\}$ with effects in $\mathcal{A}_P$, on the post-commitment state on register R; where the pointer projectors are rank-1 the restriction is vacuous and the POVM may be arbitrary, since $\mathcal{A}_P$-state equality is then operator equality. His optimal success probability is $P^{\text{opt}}_{\text{disc}}(H_p, H_e) = 1/2$ (Helstrom, by vanishing of the $\mathcal{A}_P$-restricted trace distance within the quiescent class).

Operator-algebra form. For every O in the post-decoherence pointer algebra $\mathcal{A}_P$ (the abelian von Neumann algebra generated by $\{P_k\}$),

$$\text{tr}(O \cdot \rho_{\text{select}}) \;=\; \text{tr}(O \cdot \rho_{\text{Everett}}),$$

and the operational trace distance restricted to $\mathcal{A}_P$ vanishes. In the rank-1 pointer case this is ordinary trace-norm equality on the pointer register, $\|\rho_B^{(\text{select})} - \rho_B^{(\text{Everett})}\|_1 = 0$; for rank-$r$ blocks it is equality only after restriction to $\mathcal{A}_P$. Both forms are exact within Definition 5 and become quantitative bounds outside it, with rates inherited from Theorem 25.

4 Operational Diagnostics at the Quiescence Boundary

This section gives one-way operational diagnostics for departures from quiescence, stated on the functionals of Definition 3, together with four constructions showing each diagnostic is achievable. No converse is claimed: a null reading does not certify compliance. Deletion in isolation is available for $\mathcal{C}_3$ alone; the other constructions relax a named set, and mechanistic R_1 admits no output-level diagnostic at all (Remark 26).

Reading the theorem. The witness rates use the operational metric natural to the channel opened by the violated restriction: $\mathcal{C}_1$ and $\mathcal{C}_4$ are signaling witnesses measured by trace distance on the remote marginal; $\mathcal{C}_2$ and $\mathcal{C}_3$ are Born-statistical witnesses measured by χ^2 deviation on pointer-basis frequencies. The result is a boundary witness statement on stated witness domains.

Remark 24 (Two senses of necessity, and what the constructions establish). Sense (1), *non-redundancy*: dropping R_k while keeping the remaining three restrictions on the witness domain enables an empirical channel that would otherwise be impossible. Sense (2), *uniquely diagnostic*: the witness rate ε_k (or δ_k) uniquely identifies which restriction was dropped; no other single-restriction deletion produces an identical rate.

Sense (1) in that strict form holds for $k = 3$ alone, so the theorem is weaker than individual necessity. Each remaining witness relaxes a forced set. $\mathcal{C}_1$ relaxes $\{R_1, R_4\}$: its construction Ψ_η is not a globally admissible channel and so does not preserve R_4. $\mathcal{C}_2$ relaxes $\{R_2, R_3\}$: Born-faithfulness in the rotated basis $B(\phi)$ is not Born universality in $\mathcal{A}_P$, and wherever $\delta_2 > 0$ the pointer-basis output weights differ from p, which is the failure of R_3 as Definition 5 states it. $\mathcal{C}_4$ relaxes $\{R_1, R_3, R_4\}$: the maximally mixed reference required for positivity leaves the correction nonvanishing on pointer eigenstates, and the scalar $\text{tr}(\rho^2)$ dependence is a constrained R_1 dependence. For $\mathcal{C}_1$ and $\mathcal{C}_4$ the retained restrictions are checked on pointer-eigenstate inputs while the signal is exhibited on entangled or mixed inputs of differing purity; the witness statement is the conjunction of the two, never a single claim on a common domain.

Sense (2) holds for $k \in \{1, 3\}$ and fails for $k \in \{2, 4\}$. At $k = 1$, Theorem 11(b) forbids a single-restriction witness outright: its proof consumes R_2, R_3, and R_4 and never R_1, so any law retaining those three is silent whatever its R_1 status, which is the fact Lemma 35 states from the

other side. The rate $\varepsilon_1 = \eta|\sin(\theta - \theta')|$ requires Alice-setting contrast in Bob's marginal, while an R_1-preserving map acts on the fixed state $\rho_B = I/2$, so no R_1-preserving single-restriction deletion reproduces it. At $k = 3$, δ_3 measures χ^2 deviation from Born weights, and any map preserving R_3 alongside R_4 reproduces Born weights on a spanning set and hence, by linearity, on every input. At $k = 2$, the doubly-stochastic mixing companion $M_{\phi'}$, taken at the angle fixed by $\sin^2\phi' = \frac{1}{2}\sin^2 2\phi$, reproduces δ_2 while dropping R_3 rather than R_2, and such a ϕ' exists for every ϕ. At $k = 4$, the state-dependent redescription $\mathcal{C}_4^{\mathrm{sd}}$ is formula identical to $\mathcal{C}_4$ on every input (Appendix B), so the purity-contrast rate is not uniquely diagnostic of R_4.

Theorem 25 (Operational diagnostics at the quiescence boundary)**.** *Let Φ be the run-ensemble map of a single-history selection law, and let Δ_p, Γ, Δ_{aff}, and Δ_B be the functionals of Definition 3. Each of the following implications holds, and none of the converses is asserted.*

(i) $\Delta_p(\Phi, \rho) > 0$ *at a spanning-set input implies R_3 fails.*

(ii) $\Gamma(\Phi, \rho) > 0$ *at a block-diagonal input implies R_2 fails, in the weak (diagonal-preserving) form and hence in the covariant form.*

(iii) $\Delta_{\mathrm{aff}}(\Phi; \rho, \varsigma, x) > 0$ *for some admissible ρ, ς, x implies R_4 fails.*

(iv) $\Delta_B > 0$ *across two settings of Alice implies no local trace-preserving realization $\Lambda_A \otimes I_B$ of the nonselective action exists, that is, R_L fails.*

Each diagnostic is achievable: the constructions $\mathcal{C}_3$, $\mathcal{C}_2$, $\mathcal{C}_4$, and Ψ_η of the proof realize, respectively, $\Delta_p > 0$, $\Gamma > 0$, $\Delta_{\mathrm{aff}} > 0$, and $\Delta_B > 0$, with closed-form rates on their stated witness subdomains. The constructions are existence witnesses for the functionals and are not claimed to isolate a single restriction: only $\mathcal{C}_3$ relaxes one restriction alone, and the sets relaxed by the others are named in Remark 24.

The vanishing of a diagnostic carries no information about compliance. Each rate vanishes on symmetry or degeneracy loci inside its own witness subdomain, at coinciding settings for Δ_B, at equiprobable populations for $\mathcal{C}_2$, at the uniform distribution for $\mathcal{C}_3$, and at equal test-state purities for $\mathcal{C}_4$, in none of which is the violated restriction restored. A null reading is a property of the probe.

Remark 26 (No output-level diagnostic for R_1). There is no counterpart of Theorem 25 for R_1, and the obstruction is structural rather than a gap in the constructions. R_1 constrains the per run selector, while every functional of Definition 3 reads the run-ensemble map; a selector that inspects the realized amplitudes can cancel exactly under the ensemble average, leaving all four diagnostics at zero. The sin-kernel construction of Appendix C exhibits such a law: it satisfies (β) while violating (α), so ensemble silence is compatible with mechanistic amplitude-reading. Mechanistic R_1 is therefore not decidable from output statistics at any input, which is why Lemma 35 classifies the state-dependent exits by their locality and Born fidelity rather than by a rate.

Table 2: Witness and diagnostic status per quiescence restriction. The coherence criterion diagnosing R_2 violation is established in Proposition 29; the R_4 operational non-identifiability obstruction is stated in Proposition 27; the two senses of necessity are made precise in Remark 24 preceding the theorem.

R_n	$\mathcal{C}_k$	**Boundary status**	**Sense (1): Non-Redundancy**	**Sense (2): uniquely diagnostic**	**Companion**
R_1	$\mathcal{C}_1$	Joint $\{R_1, R_4\}$; split domain	not available (Theorem 11(b))	for $\mathcal{C}_1$'s own local-realizability failure; not for mechanistic R_1 (Remark 26)	none
R_2	$\mathcal{C}_2$	Joint $\{R_2, R_3\}$	no (drops R_3 with R_2)	Proposition 29	$M_{\phi'}$
R_3	$\mathcal{C}_3$	Single	yes	yes	none
R_4	$\mathcal{C}_4$	Joint $\{R_1, R_3, R_4\}$; split domain	no (drops R_1, R_3 with R_4)	Proposition 27	$\mathcal{C}_4^{\mathrm{sd}}$

Proof by construction. Fix a two-dimensional pointer Hilbert space with projectors $P_1 = |0\rangle\langle 0|$, $P_2 = |1\rangle\langle 1|$. An incoming amplitude vector (a_1, a_2) has Born weights $p_k = |a_k|^2$.

$\mathcal{C}_1$ (drops R_1: state-dependent witness). The R_1 arc threads four sites. $\mathcal{C}_1$ here exhibits the signaling exposure of amplitude-reading selection, and Proposition 33 silences the safe subclass of Lemma 35. The quiescent class scope paragraph of Section 3 frames the dichotomy, and Section 8 closes the R_1 channel within that subclass. The construction is not used as a globally admissible physical channel; it witnesses the cost of relaxing state-independence on the stated Bell-pair witness domain.

$\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$, with pointer projectors $P_0 = |0\rangle\langle 0|$, $P_1 = |1\rangle\langle 1|$ and pointer dephasing $\mathcal{D}_B(\rho_B) = P_0\rho_B P_0 + P_1\rho_B P_1$. Alice's local rotation is fixed to the full-angle convention

$$U_A(\theta) = e^{-i\theta\sigma_y}.$$

Under the half-angle convention $e^{-i\theta\sigma_y/2}$, the same construction gives $\varepsilon_1 = \eta|\sin((\theta-\theta')/2)|$; the convention is load-bearing for the rate yet not for the qualitative conclusion, since $\varepsilon_1 > 0$ exactly when $\eta > 0$ and $\theta \neq \theta'$ under either. The input family is

$$|\Psi_\theta\rangle = (U_A(\theta) \otimes I_B)|\Phi^+\rangle, \qquad |\Phi^+\rangle = \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

Define the Bob-output witness map

$$\Psi_\eta : \mathcal{T}(\mathcal{H}_A \otimes \mathcal{H}_B) \to \mathcal{T}(\mathcal{H}_B),$$

$$\Psi_\eta(\rho_{AB}) \;=\; \mathcal{D}_B(\mathrm{tr}_A\, \rho_{AB}) \;+\; \eta\, K[\rho_{AB}],$$

where the amplitude-reading correction is

$$\sigma_{01}[\rho_{AB}] \;=\; \mathrm{tr}_A[(|0\rangle\langle 1|_A \otimes I_B)\, \rho_{AB}], \qquad K[\rho_{AB}] \;=\; \sigma_{01}[\rho_{AB}] + \sigma_{01}[\rho_{AB}]^\dagger.$$

Ψ_η is a linear witness map from the joint input state to Bob's reduced output state. It is not a B-local CPTP channel, and the witness turns on that: the output depends on off-diagonal Alice-Bob correlations invisible to ρ_B alone.

On the Bell-pair input family,

$$K\left[|\Psi_\theta\rangle\langle\Psi_\theta|\right] \;=\; \frac{1}{2}\begin{pmatrix}\sin 2\theta & \cos 2\theta\\ \cos 2\theta & -\sin 2\theta\end{pmatrix},$$

K is Hermitian and traceless, so the trace-distance rate is well defined and Bob's outputs at two Alice settings $\theta \neq \theta'$ differ by

$$\left\|\Psi_\eta(|\Psi_\theta\rangle\langle\Psi_\theta|) - \Psi_\eta(|\Psi_{\theta'}\rangle\langle\Psi_{\theta'}|)\right\|_1 \;=\; 2\,\eta\,|\sin(\theta-\theta')|, \qquad \varepsilon_1 \;=\; \tfrac{1}{2}\|\cdot\|_1 \;=\; \eta\,|\sin(\theta-\theta')|. \quad (1)$$

This is the $\mathcal{C}_1$ witness rate.

The baseline term $\mathcal{D}_B \circ \mathrm{tr}_A$ is CPTP. The added K-term is the nonlocal correlation-reading witness term and is not a B-local CPTP operation. It stays linear in ρ_{AB}; what it does is read off-diagonal Alice-Bob correlations unavailable to any B-local CPTP map. Mechanistically this violates R_1 (sense (α)): the selection rule reads correlation data encoding Alice's setting rather than acting as a SelFact with no amplitude/correlation port. Operationally this violates the B-local form of R_4: no CPTP map on B alone can reproduce θ-dependent Bob outputs from the fixed marginal $\rho_B = I/2$. The R_4 violation is a failure of B-local CPTP realizability, and not a nonlinearity of Ψ_η as a map on $\mathcal{T}(\mathcal{H}_A \otimes \mathcal{H}_B)$.

The witness is one fixed map Ψ_η at fixed η; θ enters only through the input state $|\Psi_\theta\rangle$. On pointer-eigenstate inputs $|k\rangle\langle k|_A \otimes \rho_B$, $\sigma_{01} = 0$ and K vanishes; Ψ_η reduces to the B-local CPTP dephasing $\mathcal{D}_B(\rho_B)$, so R_2 (pointer-covariance), R_3 (Born universality on pointer eigenstates), and the B-local form of R_4 hold on that restricted domain. The rate vanishes at $\eta = 0$ (state-independence restored) and at $\theta = \theta'$ (no Alice-setting contrast).

$\mathcal{C}_1$ witnesses the amplitude-reading subclass specifically ($\eta > 0$); the disjoint safe subclass is silent under Proposition 33, and that separation is the R_1 dichotomy (Lemma 35). The $\mathcal{C}_1$ violation is at the typing level (sense (α): the selection reads amplitudes and is therefore not of the SelFact form) and at the B-local-CPTP level (sense (β): no B-local channel reproduces the run-ensemble action, with rate $\varepsilon_1 > 0$ as the (β)-violation measure). For the counterexample class showing that (α)-violation does not entail (β)-violation in general, see Remark 10(2) (construction in Appendix C). The operator-level construction is indexed in Appendix B.

$\mathcal{C}_2$ (drops R_2: pointer-incovariant; coherence-generating). The selection rotates the pointer basis by a fixed angle $\phi \in (0, \pi/2)$ before projecting:

$$\mathcal{C}_2(\rho) \;=\; \sum_k \tilde{P}_k \rho \tilde{P}_k, \qquad \tilde{P}_k \;=\; U(\phi)\,P_k\,U(\phi)^\dagger.$$

$\mathcal{C}_2$ is state-independent, linear-CPTP, and Born-faithful on the *rotated* basis; it manufactures off-diagonal coherence in the original pointer basis. The χ^2-rate for distinguishing $\mathcal{C}_2$ output from the no-selection pointer marginal under a P_k-basis measurement is

$$\delta_2 \;=\; \sum_k \frac{(p_k - \tilde{p}_k)^2}{p_k} \;=\; \frac{\sin^4 2\phi}{4} \cdot \frac{(p_1 - p_2)^2}{p_1 p_2} \;>\; 0 \quad (2)$$

for $\phi \neq 0$ and non-equiprobable amplitudes. The expression for δ_2 is a χ^2 rate and diverges near zero expected probability; trace distance distinguishability remains bounded. Explicitly: $\|\mathcal{C}_2(\rho) - \mathcal{D}(\rho)\|_1 = |\sin 2\phi|\,|p_1 - p_2|$, which lies in $[0,1]$ and vanishes at $\phi = 0$ regardless of the input probabilities; the χ^2 divergence as $p_k \to 0$ is therefore a metric artifact and no physical singularity of the witness channel. *Witness domain (basis asymmetry).* The two bases must be named separately. The selector is Born-faithful in the rotated basis $B(\phi)$ and not in $\mathcal{A}_P$, while the deviation δ_2 is measured in $\mathcal{A}_P$. The witness domain is therefore: selections Born-faithful in a basis $B \neq$ pointer basis, with measurement performed in $\mathcal{A}_P$. Within that domain $\delta_2 > 0$ registers the joint departure from R_2 and R_3, and the coherence amplitude κ is what separates the two (Proposition 29).

$\mathcal{C}_3$ **(drops R_3: Born-blind).** The selection assigns uniform weight $1/n$ to each of n pointer branches regardless of a_k:

$$\mathcal{C}_3(\rho) \;=\; \frac{1}{n}\sum_k P_k \,\mathrm{tr}(\rho).$$

$\mathcal{C}_3$ is state-independent, pointer-covariant, and linear-CPTP; it reproduces Born only on the measure-zero set $p_1 = \cdots = p_n = 1/n$. The χ^2-deviation from Born statistics on a generic input is

$$\delta_3 \;=\; \sum_k \frac{(p_k - 1/n)^2}{1/n} \;=\; n\sum_k p_k^2 - 1 \;>\; 0 \tag{3}$$

whenever the input is not equiprobable; saturating at $\delta_3 = n-1$ in the fully-localized limit. The rate vanishes exactly where an amplitude-blind law is Born-correct, so $\mathcal{C}_3$ traverses in reverse the locus of Remark 19.

$\mathcal{C}_4$ **(drops R_4: nonlinear at ensemble).** $\mathcal{C}_4$ is a partial witness on its stated domain; it isolates the signaling channel opened by ensemble nonlinearity under a basis independent scalar dependence. Fix the reference $\sigma = I/n$, the maximally mixed state on the pointer algebra, with purity $\mathrm{tr}(\sigma^2) = 1/n$. The selection law is the purity-difference correction

$$\mathcal{C}_4(\rho) \;=\; \mathcal{D}(\rho) + \lambda(\mathrm{tr}(\rho^2) - \tfrac{1}{n})\,(\sigma - \mathcal{D}(\rho)), \qquad 0 < \lambda \le \frac{n}{n-1},$$

where $\mathcal{D}(\rho) = \sum_k P_k \rho P_k$ is the pointer dephasing. Writing $c(\rho) = \lambda(\mathrm{tr}(\rho^2) - \frac{1}{n})$, the map is the affine combination $\mathcal{C}_4(\rho) = (1 - c(\rho))\mathcal{D}(\rho) + c(\rho)\,\sigma$. The reference and the bound on λ are what make it a map between states: $\mathrm{tr}(\rho^2) \ge 1/n$ for every state gives $c(\rho) \ge 0$, and $\mathrm{tr}(\rho^2) \le 1$ with $\lambda \le n/(n-1)$ gives $c(\rho) \le 1$, so $c(\rho) \in [0,1]$ and the output is a convex combination of two states: positive and trace-preserving on the whole input domain. A pure reference admits no such bound: with $\mathrm{tr}(\sigma^2) = 1$ the coefficient $c(\rho)$ is non-positive for every input, the combination is an extrapolation away from $\mathcal{D}(\rho)$, and positivity fails at small pointer weight for every $\lambda > 0$. The output is diagonal in the pointer basis for every ρ and every admissible λ, so pointer-covariance (R_2) holds exactly, and not merely perturbatively. Born universality (R_3) is *not* preserved: with a mixed reference the correction does not vanish on pointer eigenstates, so $\mathcal{C}_4$ moves Born weights there. This is recorded in Theorem 25 and Table 2: $\mathcal{C}_4$ is a joint witness and not an R_4-only one. The map deliberately violates ensemble linearity (R_4) through the scalar functional $\mathrm{tr}(\rho^2)$. Because that dependence is a basis independent scalar, the witness does not read realized amplitudes in the R_1-prohibited sense, though it does depend on the state through $\mathrm{tr}(\rho^2)$; R_1 is violated in this constrained way. The companion $\mathcal{C}_4^{\mathrm{sd}}$ below shows that the rate ε_4 is not exclusive to R_4 violation, so $\mathcal{C}_4$ is a partial R_4 witness. Embedded in the Gisin-Polchinski bipartite construction, it transmits across the run-ensemble at the closed-form rate

$$\varepsilon_4 \;=\; \lambda \cdot \left|\mathrm{tr}(\rho_A^2) - \mathrm{tr}(\rho_A'^2)\right| \cdot \left\|\sigma - \mathcal{D}(\rho_A)\right\|_1 \;>\; 0 \tag{4}$$

for any two Alice settings producing distinguishable marginal purities (Bloch lengths $r_1 \neq r_2$) [17]. The Weinberg quadratic form $\rho + \lambda(\rho^2 - \rho\,\mathrm{tr}(\rho^2))$ recovers the same rate but only covaries with the pointer algebra to first order in λ; the purity-difference construction is the exact-covariance representative used here.

To make the sense-(2) failure precise, define the companion map $\mathcal{C}_4^{\mathrm{sd}}$ as follows: for each input ρ, apply the linear CPTP map $M_s(\rho) = (1-c)\,\mathcal{D}(\rho) + c\,\sigma$ where $s = \mathrm{tr}(\rho^2)$ and $c = \lambda(s - s_\sigma)$. The companion is not a single fixed linear map on the full input space: for each value of $s \in [1/n, 1]$, M_s is a fixed linear CPTP map on $\mathcal{L}(\mathcal{H})$, but the composite function $\rho \mapsto M_{\mathrm{tr}(\rho^2)}(\rho)$ is globally nonlinear in ρ, because $s = \mathrm{tr}(\rho^2)$ is quadratic in ρ. The fibers are channels for the same reason the composite maps states to states: with $\sigma = I/n$ and $\lambda \le n/(n-1)$ the coefficient $c = \lambda(s - 1/n)$ lies in $[0,1]$ for every attainable purity s, so each M_s is a convex

mixture of the dephasing channel and a fixed state. At a pure reference the coefficient is negative and the fibers are not positive maps. This composite is formula identical to $\mathcal{C}_4$ on every input (Appendix B): $\mathcal{C}_4(\rho) = M_{\mathrm{tr}(\rho^2)}(\rho)$ for all ρ. The distinction between $\mathcal{C}_4$ (described as a globally nonlinear map) and $\mathcal{C}_4^{\mathrm{sd}}$ (described as a state-dependent family) is interpretive, not operational: the two descriptions name the same map. Embedded in the same Gisin-Polchinski bipartite setting, $\mathcal{C}_4^{\mathrm{sd}}$ reproduces a run-ensemble distinguishability proportional to $\lambda\,|r_1^2 - r_2^2|$, with the same functional dependence on purity contrast as ε_4. The rate ε_4 is therefore associated with exiting quiescence through the purity-contrast channel, but not with nonlinearity alone.

Structural role of R_4. Witness function and structural function are distinct. R_4 is load-bearing in the proof of Theorem 11(b), where its linearity carries Born universality from the spanning subset of R_3 (Definition 5) to the whole simplex through the kernel argument (Appendix A), the joint quantifier being the one discussed in Section 3. $\mathcal{C}_4$ demonstrates that abandoning R_4 admits an empirical channel (ε_4), and $\mathcal{C}_4^{\mathrm{sd}}$ shows the same channel is reachable via R_1 violation, which is why the rate is not uniquely diagnostic (Proposition 27). R_4 does the structural work even when its witness is partial.

Proposition 27 (Underdetermination of the R_4 description)**.** *$\mathcal{C}_4$ and $\mathcal{C}_4^{\mathrm{sd}}$ agree on every input (Appendix B). No observation of outputs at single inputs therefore separates the description of that map as a globally nonlinear map from its description as a state-indexed family of linear ones, and no output-level or rate-level side condition on single inputs removes the ambiguity. Sense (2) fails for $k = 4$ for this reason.*

Remark 28 (What the underdetermination does and does not bound). The claim is about which description is fixed by the input-output data, and it is bounded in two ways that matter. First, both descriptions name a map that violates R_4: writing a nonlinear function as a family of linear fibers indexed by a state functional does not make the composite linear, so $\mathcal{C}_4^{\mathrm{sd}}$ is not an R_4-compliant alternative and the ambiguity is over labels, not over compliance. Second, R_4 violation is not thereby undetectable. Linearity is testable on *mixtures* rather than at single inputs: a protocol with access to prepared mixtures checks whether $\Phi(x\rho_1 + (1-x)\rho_2) = x\Phi(\rho_1) + (1-x)\Phi(\rho_2)$, and $\mathcal{C}_4$ fails this for $\rho_1 \neq \rho_2$ because $\mathrm{tr}(\rho^2)$ is quadratic. What is obstructed is the attribution of a violation to R_4 rather than to state dependence; detection is untouched. Proof in Appendix C.

M_ϕ companion for $\mathcal{C}_2$ (doubly-stochastic mixing). To see why sense (2) fails for $k = 2$, consider the doubly-stochastic mixing companion

$$M_\phi(p_1, p_2) = (\cos^2\phi \cdot p_1 + \sin^2\phi \cdot p_2,\ \sin^2\phi \cdot p_1 + \cos^2\phi \cdot p_2).$$

M_ϕ is state-independent (a fixed stochastic matrix carrying the parameter ϕ alone, with no dependence on the input amplitudes), pointer-covariant (output is diagonal in the pointer basis for all inputs), and linear stochastic (R_4 holds). It violates Born universality (R_3): the output branch probability is $\cos^2\phi \cdot p_1 + \sin^2\phi \cdot p_2 \neq p_1$ for $\phi \neq 0$ and $p_1 \neq p_2$. A direct computation on the simplex $p_1 + p_2 = 1$ gives

$$\chi^2(M_\phi) \;=\; \frac{(f_{\mathrm{out}} - p_1)^2}{p_1} + \frac{(g_{\mathrm{out}} - p_2)^2}{p_2} \;=\; \sin^4\phi \cdot \frac{(p_1 - p_2)^2}{p_1 p_2} \;=\; \delta_2.$$

The rate δ_2 therefore arises from R_3 violation alone, without any R_2 violation. The M_ϕ companion is indexed in Appendix B, and it establishes that δ_2 is not uniquely diagnostic of R_2 violation; sense (2) fails for $k = 2$ in parallel with $k = 4$.

Proposition 29 (Coherence criterion for R_2 violation)**.** *Let $\mathcal{E}$ be any single-history selection law and ρ a pointer-diagonal input. Define the pointer coherence amplitude*

$$\kappa(\mathcal{E}, \rho) \;:=\; |\langle 0|\mathcal{E}(\rho)|1\rangle|.$$

Then

$$\kappa(\mathcal{E}, \rho) > 0 \implies \mathcal{E} \text{ is pointer-incovariant } (R_2 \text{ fails}).$$

No hypothesis on Born universality, state-independence, or linearity is required. A pointer-covariant law commutes with the pointer dephasing $\mathcal{D}$, so $\mathcal{E}(\rho) = \mathcal{E}(\mathcal{D}\rho) = \mathcal{D}\mathcal{E}(\rho)$ is pointer-diagonal on a pointer-diagonal input and κ vanishes identically; the implication is the contrapositive.

The criterion is exercised by the pair of constructions of Theorem 25. The doubly-stochastic companion $M_{\phi'}$ is pointer-diagonal for every ϕ' and every input, so $\kappa(M_{\phi'}, \rho) = 0$ identically; $\mathcal{C}_2$ generates $\kappa(\mathcal{C}_2, \rho) = |p_1 - p_2| \cdot |\sin\phi\cos\phi\cos 2\phi| > 0$ for $\phi \in (0, \pi/2) \setminus \{\pi/4\}$ and $p_1 \neq p_2$. The excluded angle is not an artifact: at $\phi = \pi/4$ the rotated projectors are unbiased with respect to the pointer basis, $\mathcal{C}_2(\rho)$ is the maximally mixed state for every diagonal input, and both κ and the population shift carry no information about p. The witness angle is therefore taken away from $\pi/4$.

The two constructions share the rate δ_2 and are separated by κ alone, which is what makes κ the operative diagnostic: δ_2 measures departure from Born weights and is produced by R_2 and R_3 violations alike, while κ is produced by R_2 violation only. The two conditions are not independent, since $\delta_2 > 0$ is itself the failure of R_3 in $\mathcal{A}_P$ (Remark 24).

The coherence amplitude is defined with respect to the pointer eigenbasis $(|0\rangle, |1\rangle)$ fixed by the pointer dephasing structure; this is the canonical basis choice throughout. The proposition is stated for a two-dimensional pointer space ($n = 2$); for $n \geq 3$ it applies to each off-diagonal basis pair (i, j) separately, with κ defined as $|\langle i|\mathcal{E}(\rho)|j\rangle|$ for the chosen pair, and for higher-rank blocks to each pair of distinct blocks with κ defined as $\|P_i\,\mathcal{E}(\rho)\,P_j\|_1$. The κ-asymmetry ($\kappa(\mathcal{C}_2, \rho) > 0$ at any witness angle away from $\pi/4$, against $\kappa(M_{\phi'}, \rho) = 0$ identically) is indexed in Appendix B.

The four constructions are indexed in Appendix B. Maximality (that the quiescent class is the largest class of zero signature single-history selection laws under any extension of the witness domain) is a strictly stronger claim and is not asserted. □

5 Silent Ontology or Paid Exit

The diagnostic theorem (Theorem 25) and the equivalence case of Theorem 11 jointly classify single-history selection laws by their footprint on the post-decoherence pointer algebra: quiescence is sufficient for operational silence, and any empirically distinguishable law must violate at least one quiescence restriction.

Corollary 30 (Fork: silence within the class, cost outside it)**.** *Any single-history selection law carrying an empirical signature on $\mathcal{A}_P$ must fail at least one quiescence restriction on* Sample *or relax the exactly unitary* Flow *premise. The two axes are independent: quiescence constrains the selection law, unitarity constrains the evolution between commitments, and a model may be quiescent in* Sample *while non-unitary in* Flow*. The witness constructions exhibit four representative empirical departure modes associated with those failures on stated witness domains (each mode witnessed by the corresponding $\mathcal{C}_k$ in Theorem 25, though the specific rate of the law in question may differ from $\mathcal{C}_k$'s). Equivalently, by contraposition of Theorem 11 within the stated scope, any law satisfying all four restrictions is observationally silent. Each channel is itemized in Section 7 and isolated quantitatively by the witness construction associated with its restriction in Theorem 25. One side of the classification has a Popperian character, and it needs stating precisely. A quiescent model predicts Born statistics, pointer covariance, and unitary Flow, all of which are refutable. What is not separately testable is its distinctive actuality augmentation: relative to the same operational model without it, that augmentation is empirically underdetermined within the pointer algebra, and by theorem rather than by instrumental limitation (Remark 17). The other side is not a single category, and its members are not uniformly falsifiable (Lemma 35): empirical contact enters either by violating one of the Sample restrictions of Section 7 or by exiting the*

unitary Flow premise of Section 8, with representative costs named by the corresponding exit channels.

Proof. The forward direction is the contrapositive of Theorem 11(b): a law carrying an $\mathcal{A}_P$ signature is not silent, hence must fail at least one quiescence restriction. The reverse direction (silence under all four restrictions) is Theorem 11(b). □

Exit from quiescence without a signal. The fork is asymmetric: the converse of Corollary 30 does not hold. A single-history selection law may exit quiescence (violate R_1 at the mechanistic (α) level) while remaining operationally silent at the ensemble level, and therefore empirically indistinguishable. The safe subclass of Lemma 35 is the canonical such class (see Proposition 33, the quiescent class scope paragraph of Section 3, and the counterexample in Remark 10(2); construction in Appendix C). Violation therefore identifies the kind of cost a theory must control; the paid exit language names exposure channels, not guaranteed signals.

Remark 31 (Exit from the algebra). The fork prices departures on $\mathcal{A}_P$. A selection law may satisfy R_1 through R_4 and the exactly unitary Flow premise and still be distinguishable by a measurement outside $\mathcal{A}_P$; the environment sector is one such route (Remark 18), since the silence proof fixes the reduced pointer state alone. This is not a fifth failure mode on $\mathcal{A}_P$ but a departure from the choice of $\mathcal{A}_P$: silence is algebra-relative by construction (Definition 2), and enlarging the algebra refines the equivalence classes. Whether an extended algebra separates a given law is open; the paper settles the pointer-algebra case and states no bound beyond it.

Remark 32 (Single-shot scope of the fork). The four witness channels $\mathcal{C}_1$–$\mathcal{C}_4$ are constructed at the run-ensemble level: each probes single-shot pointer marginal statistics and run-ensemble trace distance. A single-history selection law that is empirically distinguishable only through multishot temporal correlations (for example, non-Poissonian count statistics across runs while producing Born-correct per-shot marginals) falls outside the diagnostic theorem scope by design. This paper fixes the single-shot boundary; process level and temporal-correlation extensions are outside scope.

6 No-Signaling Consistency

Theorem 11(a) covers the quiescent class. The state-dependent escape of the quiescent class scope paragraph (Section 3) must be checked for no-signaling separately: a state-dependent selection law can be nonlinear, which generically threatens superluminal (faster than light) signaling under the Gisin-Polchinski argument [17, 27]. Proposition 33 closes the obligation for the Born-faithful pointer-covariant subclass of state-dependent selections; state-dependent laws outside this subclass (non-Born-faithful or non-pointer-covariant) require case-by-case no-signaling analysis.

Proposition 33 (No-signaling under state-dependent selection, run-ensemble level)**.** *Let $\mathcal{H}_A$ be finite-dimensional of dimension n (the pointer space) and $\mathcal{H}_B$ be arbitrary (finite- or infinite-dimensional). Let ρ_{AB} be any bipartite state on $\mathcal{H}_A \otimes \mathcal{H}_B$, and let $\{P_k^\theta\}_{k=1}^n$ be a complete orthonormal projector family on $\mathcal{H}_A$ indexed by Alice's setting θ (pointer basis: $\sum_k P_k^\theta = I_A$, $P_j^\theta P_k^\theta = \delta_{jk} P_k^\theta$). Let Alice apply a local CPTP map Λ_A followed by a Born-faithful single-event selection S_θ (meaning: the run-ensemble weight assigned to outcome k by S_θ equals $\mathrm{tr}[(P_k^\theta \otimes I_B)\sigma]$ for every pointer outcome k and every post-channel state σ) satisfying pointer-covariance ($\mathcal{D}^\theta \circ \mathbb{E}_\lambda[S_\theta] = \mathbb{E}_\lambda[S_\theta] \circ \mathcal{D}^\theta$; Definition 5(ii)). Define the dephasing channel*

$$\mathcal{D}_A^\theta(\sigma) \;\coloneqq\; \sum_{k=1}^{n} (P_k^\theta \otimes I_B)\, \sigma\, (P_k^\theta \otimes I_B).$$

Then

$$\mathrm{tr}_A\big[\mathcal{D}_A^\theta((\Lambda_A \otimes I_B)(\rho_{AB}))\big] \;=\; \mathrm{tr}_A(\rho_{AB}) \;=\; \rho_B,$$

independent of θ *and of* Λ_A*, for any* $n \geq 2$ *and any* ρ_{AB}*. The Helstrom bound on distinguishing Bob's marginal across distinct settings* θ, θ' *saturates at* $1/2$*.*

Born-faithfulness in the weaker sense of correct outcome weights alone would carry the displayed conclusion only with an additional specification of the instrument realizing those weights.

Remark 34 (Nonselective ensemble statement). The driver of no-signaling is pointer-projector orthogonality ($P_j^\theta P_k^\theta = 0$ for $j \neq k$), which is definitional within the pointer-basis framework. No hypothesis on environment record orthogonality ($\langle E_j \mid E_k\rangle = \delta_{jk}$) is required; for near-orthogonal records with $|\langle E_j \mid E_k\rangle| \leq \varepsilon$ (any ε, any phase), the off-diagonal corrections to the nonselective map do not enter Bob's marginal at any order in ε, as established in Appendix B. The physical stability of the pointer basis under system-environment coupling is grounded in Zurek [31] and Joos et al. [20]; it is pointer-projector stability, not record orthogonality, that the cited decoherence theory supports.

Proof. Step 1 (Pointer-projector partial-trace identity). For any bipartite state σ on $\mathcal{H}_A \otimes \mathcal{H}_B$ and any $j \neq k$:

$$\mathrm{tr}_A\big[(P_j^\theta \otimes I_B)\,\sigma\,(P_k^\theta \otimes I_B)\big] = 0.$$

The operator $(P_j^\theta \otimes I_B)\,\sigma\,(P_k^\theta \otimes I_B)$ has A-marginal factor proportional to $|j_\theta\rangle\langle k_\theta|_A$; tracing over A returns $\langle k_\theta | j_\theta\rangle_A = \delta_{jk} = 0$ for $j \neq k$ (pointer-basis orthonormality). For pointer projectors of arbitrary rank $r \geq 1$, the same identity follows from projector orthogonality: $P_j^\theta P_k^\theta = 0$ implies $\mathrm{ran}(P_j^\theta) \perp \mathrm{ran}(P_k^\theta)$, so $\mathrm{tr}[P_j^\theta \sigma_A P_k^\theta] = 0$ for all σ_A; the rank-1 language above is the special case relevant to the pointer-decoherence setting. This identity holds for all $n \geq 2$, all ρ_{AB}, and all θ; it does not require any hypothesis on environment record overlaps $\langle E_j \mid E_k\rangle$. Established at arbitrary dimension, for non-maximally-entangled states and complex record overlaps, in Appendix B.

Step 2 (Dephasing-channel equality at the nonselective run-ensemble level). Under Born-faithfulness and pointer-covariance (Definition 5(ii)), the nonselective run-ensemble average of S_θ on any post-channel state $\sigma = (\Lambda_A \otimes I_B)(\rho_{AB})$ equals

$$\mathbb{E}[S_\theta(\sigma)] = \sum_k (P_k^\theta \otimes I_B)\,\sigma\,(P_k^\theta \otimes I_B) \;+\; \sum_{j \neq k} \alpha_{jk}\,(P_j^\theta \otimes I_B)\,\sigma\,(P_k^\theta \otimes I_B),$$

where the coefficients α_{jk} encode any residual environment record overlap structure (including the near-orthogonal case $|\langle E_j \mid E_k\rangle| \leq \varepsilon$). Taking the partial trace over A and applying Step 1 to every off-diagonal term ($j \neq k$):

$$\mathrm{tr}_A\big[\mathbb{E}[S_\theta(\sigma)]\big] = \mathrm{tr}_A\big[\mathcal{D}_A^\theta(\sigma)\big] \;+\; \sum_{j \neq k} \alpha_{jk} \cdot 0 = \mathrm{tr}_A\big[\mathcal{D}_A^\theta(\sigma)\big].$$

The off-diagonal terms vanish at the level of Bob's marginal for *any* α_{jk}, independently of record-overlap structure. The conclusion is at the Bob marginal level only: the full-state nonselective map $\mathbb{E}[S_\theta(\sigma)]$ retains the off-diagonal coherent crossterms $\sum_{j \neq k} \alpha_{jk}(P_j^\theta \otimes I_B)\sigma(P_k^\theta \otimes I_B)$ and is not equal to $\mathcal{D}_A^\theta(\sigma)$ in general; a pointer-diagonal unitary $U_\theta = \sum_k e^{i\phi_k(\theta)} P_k^\theta$ is the canonical counterexample (pointer-covariant, CPTP, with $\alpha_{jk} = e^{i(\phi_j - \phi_k)} \neq 0$). The full-state identity $\mathbb{E}[S_\theta(\sigma)] = \mathcal{D}_A^\theta(\sigma)$ holds only in the orthogonal record limit ($\alpha_{jk} = 0$). Born-faithfulness is load-bearing for the *selective* per-outcome content of clause (b): it pins the per-outcome conditional weight to $p_k = \mathrm{tr}[(P_k^\theta \otimes I_B)\sigma]$, which makes the Helstrom statement at the selective level well-defined; without Born-faithfulness, the selective weights could be $q_k(\theta) \neq p_k$ and Bob's marginal would carry residual θ-dependence through trace-non-preservation on the A-factor.

Step 3 (Trace-preservation and Helstrom). The dephasing channel $\mathcal{D}_A^\theta$ is trace-preserving on the A-factor (textbook; see, e.g., [26]), and Λ_A is CPTP by hypothesis; hence

$$\rho_B(\theta) = \mathrm{tr}_A\big[\mathcal{D}_A^\theta((\Lambda_A \otimes I_B)(\rho_{AB}))\big] = \mathrm{tr}_A\big[(\Lambda_A \otimes I_B)(\rho_{AB})\big] = \mathrm{tr}_A(\rho_{AB}) = \rho_B,$$

independent of θ and Λ_A. Since $\rho_B(\theta) = \rho_B(\theta')$ as operators, the Helstrom bound [18] gives

$$P_{\text{disc}}^{\text{opt}} = \tfrac{1}{2} + \tfrac{1}{2}\, D(\rho_B(\theta),\, \rho_B(\theta')) = \tfrac{1}{2},$$

where $D(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1$ is the trace distance (Nielsen-Chuang convention). Throughout this paper, D denotes trace distance and $\|\cdot\|_1$ denotes trace norm; the equivalent trace-norm form of the Helstrom bound is $P_{\text{disc}}^{\text{opt}} = \frac{1}{2} + \frac{1}{4}\|\rho - \sigma\|_1$. □

7 Escape Routes

Every ontology that reproduces standard QM and hopes to carry an observable signature must either fail at least one quiescence restriction on Sample (Definition 5) or relax the exactly unitary Flow premise. The active rivals partition along this axis. For each program we give the primary exit (which restriction is dropped or premise relaxed) and the empirical, operational, or ontological price it triggers.

The exit map as a construction question. Each route by which a collapse or selection model could acquire empirical content [4, 23] is keyed here to the restriction whose failure it diagnoses. Read forwards, the map answers a question a model-builder can act on: what further structure must a framework introduce before its actuality fact becomes physical rather than ontological, that is, before Truthmaker(Actual(k, e)) acquires a footprint on the accessible algebra? The admissible answers are enumerable, and each carries a stated cost and a channel. Altered outcome probabilities exit through R_3, at the χ^2 rates δ_2 and δ_3. New pointer coherence exits through R_2, detected by the amplitude κ (Proposition 29). Modified dynamics exits the Flow premise, at the heating and emission rates of Section 8. Nonlinear dependence on the state exits through R_4, at the purity-contrast rate ε_4, testable on mixtures (Proposition 27). Record structure enters through the overlaps α_{jk}, where the marginal identity of Proposition 33 holds throughout and the full-state identity only in the orthogonal limit. Temporal correlation across runs is outside the single-shot scope fixed here (Remark 32) and is the open entry on the list. Direct coupling of the selection label to an accessible observable is excluded by the typing of k^* (Definition 1); a framework that wants it must abandon that typing and inherits the Gisin-Polchinski constraint on the amplitude-reading branch (Lemma 35). A further entry leaves the accessible algebra rather than the quiescent class: a law may satisfy every restriction and exactly unitary Flow while carrying structure outside $\mathcal{A}_P$ (Remark 31).

The entries are not claims of individual necessity, which Theorem 25 does not establish (Remark 24). They are the places where structure can enter and the price charged at each. A framework is located on this map by naming which entry it takes. The enumeration covers the operationally live exit programs; it is not exhaustive of the scoped hatches listed in Remark 18.

The exposure channels of Section 5 are the ones named below: each exit opens a specific empirical departure mode whose rate is law-dependent and may vanish on a non-empty subclass (e.g., the safe subclass of Lemma 35 under R_1).

Typology by restriction

The exits are organized below by the restriction relaxed (scoped hatches in Remark 18), each with its falsifiability verdict and the structural reason behind it.

R_1 **state-dependent selection.** *Not falsifiable within the Born-faithful pointer-covariant class.* State-independence (Definition 5(i)) is the relaxed premise.

Lemma 35 (R_1 dichotomy)**.** *Any state-dependent (R_1-violating) single-history selection law falls into one of three subclasses:*

- Safe subclass: *Born-faithful and pointer-covariant, state-dependent. The run-ensemble action agrees with $\mathcal{D}_A^\theta$ on Bob's marginal (Proposition 33); operationally silent at the ensemble level despite mechanistic (α)-violation, hence empirically indistinguishable from quiescence.*
- Signaling subclass: *amplitude-reading laws whose induced run-ensemble map on the joint system is not of the local form $\Lambda_A \otimes I_B$. Opens the Gisin-Polchinski signaling channel ($\mathcal{C}_1$ of Theorem 25, rate $\varepsilon_1 > 0$); excluded by no-signaling [17, 27].*
- Locally deviant subclass: *laws that fail Born-faithfulness or pointer-covariance while remaining genuinely local and trace-preserving on A. These do not signal, since any map of the form $\Lambda_A \otimes I_B$ with Λ_A trace-preserving leaves Bob's marginal invariant whatever it does to Born weights or pointer coherence. They are constrained instead by the local channels of Theorem 25: a Born-weight departure registers as $\delta > 0$ and a coherence departure registers as $\kappa > 0$ (Proposition 29).*

Locality, not Born-faithfulness, is what separates signaling from silence. The exclusion of the signaling subclass is therefore an exclusion of nonlocal induced maps, and R_1 exits that survive no-signaling are absorbed either into operational silence (safe subclass) or into a locally testable Born or coherence departure (locally deviant subclass).

R_2 **basis drift** ϕ**.** *Not falsifiable in current channels.* Pointer-covariance (Definition 5(ii)) is the relaxed premise; the basis-asymmetry witness $\mathcal{C}_2$ activates (Theorem 25), but the pointer basis is underidentified by ambient decoherence in realistic environments, so the witness domain is not isolated empirically. Joint diagnostic closure via (δ_2, κ) is conditional (Proposition 29).

R_3 **Born-deviation** ε**.** *Logical boundary witness, not a near-term empirical falsifier.* Born universality (Definition 5(iii)) is the relaxed premise. $\mathcal{C}_3$ (Theorem 25) demonstrates that dropping R_3 admits non-Born statistics on the simplex at rate $\delta_3 > 0$; the empirical test is calibration-dominated in current precision experiments, so $\mathcal{C}_3$ functions as a logical boundary witness and carries no falsifier recommendation.

Flow premise (Lindblad γ). *Falsifiable; recommended primary channel.* The relaxed premise is exactly unitary Flow (linear CPTP at the integrated channel preserved; R_4 holds). Operational signature: anomalous heating and spontaneous emission, with the ratio fixed by γ at each mass scale (the joint independent-channel falsifier of Remark 36). Constraint family: xenon X-ray [30]; free-fall force noise [8, 9]; cosmological heating (Appendix D).

Scope: falsifiability is asymmetric. A positive heating signal falsifies $\gamma = 0$ (quiescent BIT), but does not uniquely localize the mechanism to single-history selection: the heating channel is shared with CSL, GRW, Diosi-Penrose, and generic environmental decoherence. The joint heating/emission ratio narrows the active mechanism to position-coupling Lindblad models within the locked convention of (6), but not specifically to BIT's selection architecture. A negative signal $\gamma \leq$ bound is consistent with quiescent BIT and with many other null hypotheses. Falsification here disconfirms the $\gamma = 0$ prediction; uniquely identifying any one alternative requires further structural assumptions.

Non-CPTP / retrocausal. *Not falsifiable in current channels.* CPTP at the run-ensemble is relaxed via future-boundary non-Markovianity; final-boundary averaging reproduces $\mathcal{D}_A^\theta$ on the pointer marginal (Proposition 33, safe class). Operationally silent in current channels.

Environment-assisted (multishot temporal). *Conditional falsifiability, outside the diagnostic theorem witness domain.* Quiescent on every single shot (Born-correct per-shot marginals) but distinguishable through multishot temporal correlations only (non-Poissonian count statistics across runs); outside the diagnostic theorem scope by design (Remark 32). Falsifiability requires a multishot cross-run protocol; optical-lattice clock comparisons and precision photon-counting stability at the 10^{-18} level are the natural experimental targets, though converting a stability figure into a bound requires a stochastic model of the postulated cross-run process and a transfer function from its correlation spectrum to the clock observable, neither of which is supplied here.

8 Falsifiability: Lindblad Flow Exit and Experimental Bounds

Framing. A falsifiable extension of BIT does not deform the theorem: the Lindblad family (the most general Markovian CPTP open-system generator; full entry in Key terms, Section 1) exits its scope by relaxing the exactly unitary Flow premise while preserving linear CPTP evolution, so the departure is from unitary Flow, not from the CPTP property of Sample itself. The one-parameter family S_γ with $\gamma \geq 0$ makes this precise: at $\gamma = 0$, S_0 is quiescent and BIT applies; at $\gamma > 0$, S_γ exits via the Flow premise, producing a measurable decoherence-and-heating channel. The experimental bounds in this section constrain the Lindblad Flow exit model at $\gamma > 0$; the quiescent single-history selection ontology remains at $\gamma = 0$ and is operationally silent by BIT.

The Lindblad Flow exit. The Lindblad Flow exit adds a single dissipator to the unitary von Neumann equation. With X denoting the mass-weighted position operator and $\gamma \geq 0$ a coupling constant, the evolution is governed by

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] - \gamma\,[X, [X, \rho]], \qquad \gamma \geq 0. \tag{5}$$

The dissipator $\gamma[X, [X, \rho]] = \gamma(X^2\rho + \rho X^2 - 2X\rho X)$ has jump operator $L = \sqrt{2\gamma}\,X$ and a positive semidefinite Kossakowski matrix; the resulting channel is linear CPTP at every time $t \geq 0$. At $\gamma = 0$ the equation reduces to the von Neumann equation exactly, and BIT quiescence is recovered. For $\gamma > 0$, the dissipator produces additional decoherence in the X-eigenbasis and anomalous energy injection at rate proportional to γ.

The Lindblad exit parameter γ is a dynamical dissipation rate, independent of the run-ensemble witness rate $\varepsilon_4 = \lambda|\operatorname{tr}\rho_A^2 - \operatorname{tr}\rho_A'^2|$ of $\mathcal{C}_4$; the two quantities diagnose BIT-exit at different operational levels and are not interchangeable.

Per-channel falsifiability. The four candidate empirical channels are not naturally unified into a single model. Three correspond to Sample-restriction exits; the Lindblad channel instead relaxes exact unitary Flow while preserving linear CPTP evolution. Their experimental signatures are operationally distinct.

Exit from the Flow premise (Lindblad γ): Foundations-clean. This is an exit from the Flow premise: linear CPTP at the run-ensemble (R_4) holds for all $\gamma \geq 0$, and the departure is from exactly unitary Flow alone. The Lindblad form is CPTP and linear for all $\gamma \geq 0$, recovers quiescence at $\gamma = 0$, and inherits no-signaling from its local linear CPTP structure.[1]

[1] Proof: $\operatorname{tr}_A[(\Phi_A^\gamma \otimes I_B)(\rho_{AB})] = (\operatorname{tr}_A \circ \Phi_A^\gamma \otimes I_B)(\rho_{AB}) = (\operatorname{tr}_A \otimes I_B)(\rho_{AB}) = \rho_B$, by trace-preservation of Φ_A^γ on the A-factor ($\operatorname{tr}_A \circ \Phi_A^\gamma = \operatorname{tr}_A$). This argument is self-contained and distinct from the Born-faithful state-dependent case of Proposition 33.

Because $\mathcal{C}_4$ is a partial witness (sense (2) fails; see Section 4), a measured heating rate constrains the Lindblad Flow exit without distinguishing it from an R_1 exit, and telling the two apart requires a channel-specific experimental design. The Lindblad exit from the Flow premise is the recommended primary falsifiable channel.

Where a cost would hide. If a single-history selection signature is nonzero but universal, it is unlikely to appear first as an ordinary Born-statistical anomaly. Its natural hiding places are weak localization, heating, spontaneous radiation, mechanical force noise, or loss of macroscopic coherence. These channels are especially constraining because small couplings accumulate over long times, large masses, many particles, or cosmological baselines. Thus the quiescent point $\gamma = 0$ is structurally special: zero coupling remains silent, whereas any nonzero universal Lindblad-type coupling is constrained by the same amplification mechanisms that bound collapse models.

Experimental bounds on γ. If single-history selection couples to position with nonzero strength γ, the master equation (5) generates anomalous heating, spontaneous radiation, and coherence loss at rate proportional to γ. The closed-form momentum-diffusion prediction of the master equation is $d\langle p^2\rangle/dt = 2\gamma\hbar^2$, derived from the double-commutator dissipator via $\mathrm{Tr}[p^2 \cdot \gamma(2X\rho X - X^2\rho - \rho X^2)] = 2\gamma\hbar^2$ using $[X, p] = i\hbar$ (canonical commutation; see Breuer and Petruccione, §3.3.3 [6]). This gives an energy injection rate $Q = \gamma\hbar^2/M$ at nucleon mass scale; for a composite body of nucleon number A with collective position coupling, the rate is enhanced by A^2. The strongest current bounds (xenon spontaneous X-ray, free-fall mechanical force noise, cosmological heating) are collected in Appendix D under the locked convention (6); that landscape is expected to evolve as detector sensitivity improves. No confirmed signal has been detected in any channel; $\gamma = 0$ (quiescent BIT) remains consistent with all current data.

Remark 36 (Joint heating-emission ratio: independent-channel falsifier). The dissipator $\gamma[X, [X, \rho]]$ governs both momentum diffusion (heating) and spontaneous photon emission through the same coupling γ. A joint measurement of both channels therefore constrains γ from two independent observables. The ratio (heating rate)/(spontaneous emission rate) is a fixed prediction of the Lindblad exit at each mass scale; a mismatch between the γ extracted from each channel would falsify the Lindblad exit hypothesis specifically, but not generic decoherence.[2] This joint-channel ratio discriminates the position-coupling Lindblad exit from rival exits (CSL with coupling r_C, Diosi-Penrose gravitational decoherence, generic environmental decoherence) within the linear-response regime and the locked mass-scale convention of (6). It does not repair sense-(2) of the $\mathcal{C}_4$ witness: by Proposition 27, no operational closure of the R4 channel is available, and the heating-emission ratio leaves Proposition 33 and clause (b) of Theorem 11 intact while sharpening the empirical discriminator carried by the $\mathcal{C}_4$ paragraph of Theorem 25. The ratio is fixed here as a prediction of the Lindblad Flow exit; the paper does not evaluate it against current experimental sensitivity, and Appendix D records the two channels the joint constraint would draw on rather than a numerical bound on the ratio.

Scope notes. Three constraints govern this section. First, a γ-bound constrains the Lindblad Flow exit; distinguishing it from R1-type exit requires additional channel-specific experimental design. Second, the state-dependent channel (R1) requires a new no-signaling proof before it can serve as a falsifiable extension. Third, the Everett-with-label comparison construct remains fixed at $\gamma = 0$; no extension of the comparison construct is required or appropriate. The fork corollary (Corollary 30) is satisfied at $\gamma > 0$ through its second disjunct: the Flow premise is relaxed while Sample may remain quiescent, since R_1 through R_4 constrain the selection law

[2] The explicit spontaneous emission rate under position-coupling Lindblad is derived in Collett and Pearle [11]; the heating-to-emission ratio is fixed by γ and the nuclear form factor at each mass scale.

and say nothing about the evolution between commitments. The exit from the Flow premise is operationally viable only in the open interval $0 < \gamma < \gamma_{\max}$, with the upper bound $\gamma_{\max}$ set by current experimental sensitivity (the instrumental regime of Remark 17). At $\gamma = 0$ the exit restores the unitary Flow premise and BIT applies; above $\gamma_{\max}$ the model is excluded by experiment. The paper fixes the channel structure and the conversion convention that determine $\gamma_{\max}$ (Appendix D), and assigns it no numerical value: the bound follows from whichever limit currently leads the emission channel and moves with it.

Operational null hypothesis: the Helstrom discriminator. A quiescent selection law predicts $\varepsilon = 0$ on $\mathcal{A}_P$ (Remark 23); a confirmed $P_{\text{disc}}^{\text{opt}} = 1/2 + \varepsilon$ for $\varepsilon > 0$ constitutes a direct BIT-exit signal, localizing the exit to at least one violated restriction via the two-senses anatomy of Theorem 25. The collapse-model bounds of Appendix D are not measurements of ε: each constrains a parameter of a specified model through a model-specific observable, and the relation to the Lindblad exit runs through the normalization fixed at (6), which is a stated long-wavelength convention for a particular model class and not a model-independent conversion.

9 Single-History Selection in the Quiescent Class as Silent Ontological Structure

This section reads the silent ontology positioning through the silence result of Section 3; the quiescence restrictions delimit the class as set out in Section 2.

What BIT decides. BIT does not decide whether the single-history selection fact is real. The fork (Corollary 30) shows that any single-history selection fact satisfying the four quiescence restrictions is operationally silent; it does not show that the selection fact is unreal or that B-theoretic deflationism is vindicated. The classification result is this: single-history selection structure, if it belongs to the quiescent class, is ontological structure that carries no operational signature in the accessible pointer algebra.

Populating the class. The class is not exhausted by the identity realizer. A global-boundary selection of one branch, of the kind developed in single-world quantum mechanics [22–24], is a candidate member, with membership conditional on R_2 and R_4 for the global-to-local marginal projection. Verifying R_4 requires a structural argument about the projection map rather than an operational test at the pointer-marginal level: per Proposition 27, no output state measurement on any single input fixes whether a departure is attributed to nonlinearity or to state dependence, so the R_4 case is qualitatively harder to close than the R_2 case; a mixture-preservation test detects a departure but does not settle the attribution, and the projection map's compliance remains a structural question. The identity realizer satisfying all four restrictions strictly is a member by the definition and the proof of Lemma 37.

Identity-on-populations is not triviality. Within the quiescent class, the run-ensemble population map is the identity for every member (Lemma 37), because that identity is exactly what the four restrictions force. The identity is what the theorem asserts; it is no sign of an empty class. The non-triviality of a quiescent inhabitant is located at the per run level: the single-history commitment selects one branch as actual, and that selection fact is a substantive predicate not reducible to any expectation value. Whether actuality here is an intrinsic thisness of the branch (a monadic primitive, haecceistic) or a relational property conferred by the single-history selection law is not resolved by BIT; this is the truthmaker gap the realizing framework must close.

Truthmaker gap. BIT is conditional on the realizing framework's substantive commitment that Selected(k^*, e) lifts to Actual(k^*, e), i.e., that the framework supplies Truthmaker(Actual(k^*, e)). That per run truthmaker for Actual(k, e) is exactly the measurement problem burden, and it is the realizing framework's, not BIT's. BIT conditions on such a framework and classifies the operational footprint of any resulting selection fact under R_1–R_4. The fork (Corollary 30) is the full statement of what BIT delivers: within the quiescent class the framework's truthmaker is operationally invisible on $\mathcal{A}_P$; outside the quiescent class the framework pays through one of the named exit channels of Sections 7 and 8.

A supplement of this kind leaves the operational statistics untouched, a point that appears in several particular forms: the gap between an improper and a proper mixture [12], the failure of decoherence to deliver a definite outcome [1, 19, 28], a theorem on the reach of the superposition principle [3], and, for the Bohmian case, a proof that a Born-distributed corpuscle is undetectable, with the limitation irreducible rather than technological [14]. The classification here holds for a class named by restrictions rather than for a single theory, and carries the boundary and the failure of the converse.

Two nearby results are distinct from it. Theorem 11 is not a corollary of the no-extension theorem of [10]: that result excludes extensions improving predictive power over quantum theory, whereas the quiescent class consists of laws reproducing quantum predictions exactly, and the question here is which of them carry a pointer-algebra signature, not whether any improves prediction. Nor is the method that of [7], which characterizes interpretations by a uniqueness theorem over a chosen determinate set; the restrictions here are conditions on a selection law's action, and the object classified is its operational footprint rather than its determinate observables.

The truthmaker gap is the exact residual metaphysical burden after the operational channel is closed, and that residual is the philosophical payoff of the theorem.

Silence within the class. Within the quiescent class, Proposition 29's criterion cannot fire: pointer-covariance forbids new pointer coherence, so $\kappa = 0$ throughout. The coherence criterion detects R_2 violation and so applies only outside the class. Silence on $\mathcal{A}_P$ is a statement about what the algebra resolves, and carries no implication that the selection fact is redundant: Actual(k, e) is typed apart from every operational predicate (Definition 1, Remark 20), so inertness in the accessible algebra and substantiveness in the ontology are independent by construction. Operational silence is a classification result and no deflationary argument. Any single-history ontology that hopes to carry a pointer-algebra operational signature must exit the quiescent class through a paid channel of Section 7, and exit alone does not suffice (Corollary 30).

10 Discussion

What the classification delivers. Quiescent (under R_1–R_4) single-history selection structure is classified as silent ontological structure: substantive in the realizing framework's ontology and inert in the accessible algebra (Section 9). BIT is paradigm-neutral on which framework realizes the single-history selection fact, and does not decide whether that fact is Actual in the realizing-framework sense; the nonclaims are stated in full at Section 1.

What the result does is separate ontic commitment from empirical content. A quiescent single-history selection fact may be part of an ontology, but it is not an additional physical observable. Conversely, a single-history selection fact meant to be empirically detectable must leave the quiescent class by changing weights, coherence, covariance, or dynamics. In this sense, the theorem gives a boundary on what "reality of the single-history selection fact" can mean inside standard quantum mechanics.

A realizing framework supplies the truthmaker for the selection fact. Phenomenological apparatus locating the selection fact at an observer's record-formation boundary lies outside the

scope of this paper and is not load-bearing for BIT.

BIT separates actuality from causal efficacy. A selected branch need not differ from unselected branches by any additional observable, energy contribution, or dynamical term. Its difference is not dynamical but indexical. The single-history selection fact is not a mechanism added to quantum theory; it is an ontological index carried silently by one realized history.

Cost of the silent ontology classification. The price of silent structure is that the ontology carries no falsifier separate from the operational model it augments; see Section 9 for the classification result and Section 7 for the empirical channels that bound the active rivals.

The quiescent class and falsifiability. The fork corollary (Corollary 30) identifies one class exactly and leaves its complement open. Every member of the quiescent class is operationally silent on $\mathcal{A}_P$ by Theorem 11, and so its actuality augmentation is not separately testable there as a positive dynamical hypothesis. The complement is not the class of laws that can carry a signature: exiting quiescence is necessary for empirical content on $\mathcal{A}_P$ and does not supply it, as the Born-faithful pointer-covariant state-dependent subclass and the locally deviant subclass both show (Lemma 35). Falsifiability enters where a law exits quiescence or modifies Flow *and* the exit opens a channel, which the paper exhibits case by case rather than in general. On the single-shot pointer algebra no increase in experimental sensitivity changes this: the saturation is a theorem, not an instrumental limit. The multishot temporal setting is outside that scope and carries no such saturation result (Remark 32). Within the quiescent class the actuality augmentation is not separately discriminable from the operational baseline on $\mathcal{A}_P$, while the complete operational model remains falsifiable; BIT distinguishes that class from the falsifiable extensions (Sample-restriction exits and exits from the Flow premise of Section 7). A single-history selection program that insists on empirical traction must exit quiescence; a program content with silence pays no empirical price but forfeits observational discriminability, in principle and not merely in practice.

Verification. Formal verification is collected in Appendix B.

References

[1] S. L. Adler. "Why decoherence has not solved the measurement problem: a response to P. W. Anderson". In: *Studies in History and Philosophy of Modern Physics* 34 (2003), pp. 135–142. DOI: 10.1016/S1355-2198(02)00086-2.

[2] S. L. Adler. "Lower and upper bounds on CSL parameters from latent image formation and IGM heating". In: *Journal of Physics A: Mathematical and Theoretical* 40 (2007), pp. 755–763. DOI: 10.1088/1751-8113/40/4/011.

[3] A. Bassi and G. C. Ghirardi. "A general argument against the universal validity of the superposition principle". In: *Physics Letters A* 275 (2000), pp. 373–381. DOI: 10.1016/S0375-9601(00)00612-5.

[4] A. Bassi et al. "Models of wave-function collapse, underlying theories, and experimental tests". In: *Reviews of Modern Physics* 85 (2013), pp. 471–527. DOI: 10.1103/RevModPhys.85.471.

[5] J. Bigelow. "Presentism and properties". In: *Philosophical Perspectives* 10 (1996), pp. 35–52.

[6] H.-P. Breuer and F. Petruccione. *The Theory of Open Quantum Systems*. Oxford University Press, 2002.

[7] J. Bub and R. Clifton. "A uniqueness theorem for interpretations of quantum mechanics". In: *Studies in History and Philosophy of Modern Physics* 27 (1996), pp. 181–219. DOI: 10.1016/1355-2198(95)00019-4.

[8] M. Carlesso, A. Vinante, and A. Bassi. "Improved bounds on collapse models from rotational noise of LISA Pathfinder". In: *arXiv preprint* (2025). arXiv:2501.08971.

[9] M. Carlesso et al. "Experimental bounds on collapse models from gravitational wave detectors". In: *Physical Review D* 94 (2016), p. 124036. DOI: 10.1103/PhysRevD.94.124036.

[10] R. Colbeck and R. Renner. "No extension of quantum theory can have improved predictive power". In: *Nature Communications* 2 (2011), p. 411. DOI: 10.1038/ncomms1416.

[11] M. J. Collett and P. Pearle. "Problems with the Newton-Schrödinger equations". In: *Foundations of Physics* 33 (2003), pp. 1495–1541. DOI: 10.1023/A:1026048530567.

[12] B. d'Espagnat. *Conceptual Foundations of Quantum Mechanics*. 2nd. Reading, MA: W. A. Benjamin, 1976.

[13] S. Donadi et al. "Underground test of gravity-related wave function collapse". In: *Nature Physics* 17 (2021), pp. 74–78. DOI: 10.1038/s41567-020-1008-4.

[14] D. Dürr, S. Goldstein, and N. Zanghì. "Quantum equilibrium and the origin of absolute uncertainty". In: *Journal of Statistical Physics*. Vol. 67. New York: Springer, 1992, pp. 843–907. DOI: 10.1007/BF01049004.

[15] H. Everett. ""Relative state" formulation of quantum mechanics". In: *Reviews of Modern Physics* 29.3 (1957), pp. 454–462. DOI: 10.1103/RevModPhys.29.454.

[16] D. J. Fixsen et al. "The cosmic microwave background spectrum from the full COBE FIRAS data set". In: *The Astrophysical Journal* 473 (1996), pp. 576–587. DOI: 10.1086/178173.

[17] N. Gisin. "Stochastic quantum dynamics and relativity". In: *Helvetica Physica Acta* 62 (1989), pp. 363–371.

[18] C. W. Helstrom. *Quantum Detection and Estimation Theory*. New York: Academic Press, 1976.

[19] E. Joos and H. D. Zeh. "The emergence of classical properties through interaction with the environment". In: *Zeitschrift für Physik B* 59 (1985), pp. 223–243. DOI: 10.1007/BF01725541.

[20] E. Joos et al. *Decoherence and the Appearance of a Classical World in Quantum Theory*. 2nd. Berlin: Springer, 2003. DOI: 10.1007/978-3-662-05328-7.

[21] A. Kent. "Against many-worlds interpretations". In: *International Journal of Modern Physics A* 5 (1990), pp. 1745–1762. DOI: 10.1142/S0217751X90000805.

[22] A. Kent. "One world versus many: the inadequacy of Everettian accounts of evolution, probability, and scientific confirmation". In: *Many Worlds? Everett, Quantum Theory, and Reality*. Ed. by S. Saunders et al. Oxford University Press, 2010. DOI: 10.1093/acprof:oso/9780199560561.003.0012.

[23] A. Kent. "Solution to the Lorentzian quantum reality problem". In: *Physical Review A* 90 (2014), p. 012107. DOI: 10.1103/PhysRevA.90.012107.

[24] A. Kent. "Lorentzian quantum reality: postulates and toy models". In: *Philosophical Transactions of the Royal Society A* 373 (2015), p. 20140241. DOI: 10.1098/rsta.2014.0241.

[25] N. Markosian. "A defense of presentism". In: *Oxford Studies in Metaphysics* 1 (2004), pp. 47–82.

[26] M. A. Nielsen and I. L. Chuang. *Quantum Computation and Quantum Information*. Cambridge: Cambridge University Press, 2000. DOI: 10.1017/CBO9780511976667.

[27] J. Polchinski. "Weinberg's nonlinear quantum mechanics and the EPR paradox". In: *Physical Review Letters* 66 (1991), pp. 397–400. DOI: 10.1103/PhysRevLett.66.397.

[28] M. Schlosshauer. "Decoherence, the measurement problem, and interpretations of quantum mechanics". In: *Reviews of Modern Physics* 76 (2005), pp. 1267–1305. DOI: 10.1103/RevModPhys.76.1267.

[29] S. Weinberg. "Testing quantum mechanics". In: *Annals of Physics* 194 (1989), pp. 336–386. DOI: 10.1016/0003-4916(89)90276-5.

[30] XENON Collaboration. "Challenging Spontaneous Quantum Collapse with the XENONnT Dark Matter Detector". In: *Physical Review Letters* (2026). arXiv:2506.05507. DOI: 10.1103/2jm3-4976.

[31] W. H. Zurek. "Decoherence, einselection, and the quantum origins of the classical". In: *Reviews of Modern Physics* 75 (2003), pp. 715–775. DOI: 10.1103/RevModPhys.75.715.

A Proof Kernel

The Branch Indexical Theorem rests on one linear-algebra step.

Lemma 37 (Identity from universal fix)**.** *Let M be a real $n \times n$ matrix. If $Mv = v$ for every v in the standard probability simplex on $\mathbb{R}^n$ (equivalently, for a spanning set of vectors), then $M = I$.*

Proof. The probability simplex contains the standard basis vectors $e_1, \ldots, e_n$ (through its vertices). The condition $Me_i = e_i$ for each i identifies column i of M with e_i, so $M = I$. □

Use in BIT. By the linear CPTP restriction (Definition 5(iv)), the population map M induced by any quiescent Sample map is linear; by Born universality (Definition 5(iii)), M fixes each vertex of the pointer simplex. Lemma 37 concludes the population map is the identity, which injects zero energy through observables in the pointer algebra ($H \in \mathcal{A}_P$; Appendix B F3), and, for the dephasing channel $\mathcal{D}$ specifically, through any pointer-commuting observable (Appendix B C8), and creates no new pointer coherence, hence no quiescent class signature. The physical content of Theorem 11(b) lies entirely in the four modeling restrictions of Definition 5; the kernel step is elementary.

Verification. Formal verification of Lemma 37 and the corollaries that ride on it (energy injection F3, Helstrom saturation F6) is collected in Appendix B.

B Formal Verification

Verification-code glossary. Body references use check codes Fk, Ck, Tk. Each code is decoded once here; the artifact that establishes each code is named in Table 4.

Table 3: Verification-code glossary.

Code	Verifies
F1	Forcing-lemma kernel: a matrix fixing every probability vector is the identity (Lemma 37).

(continued on next page)

(Table 3, continued)

Code	Verifies
F2	Born-universality is the load-bearing hypothesis: a non-identity stochastic map fixes one probability vector while moving another.
F3	Identity population map injects zero energy through observables in the pointer algebra ($H \in \mathcal{A}_P$).
F4	Loud-horn witness: a coherence-creating map exits the selection class by non-pointer-covariance, confirming the dichotomy is non-vacuous.
F5	Forcing-lemma robustness: fixing the point masses suffices, and an affine Born-universal map fixing the point masses fixes the whole simplex (robustness of Lemma 37).
F6	Operational silence of the identity population map at fixed post-decoherence pointer marginal.
F7	Dephasing channel identity on the pointer marginal: exact no-signaling on Bob's marginal under selection on A, $\mathrm{tr}_A[\mathcal{D}_A(M)] = \mathrm{tr}_A(M)$ for generic M (Proposition 33).
F8	Signed energy-rate separator: the secular warming rates of the spontaneous-localization family are strictly positive against zero for the keyed present (distinguishing corner against GRW/CSL/DP).
F9	Bridge from population identity to $\mathcal{A}_P$-state silence: $R_3 + R_4$ ($M = I$) alone $\Rightarrow$ $\mathrm{tr}(O\Phi(\sigma)) = \mathrm{tr}(O\sigma)$ for all $O \in \mathcal{A}_P$ on pointer-diagonal input, since agreement of block weights already forces $\mathcal{A}_P$ equality; R_2 is not a premise of this step and instead supplies coherence-neutrality and, at rank-1 pointer projectors, the lift to full operator equality $\Phi(\sigma) = \sigma$.
C1	Fiber-counting bound: $\prod_k \lceil 1/a_k \rceil \geq \lceil 1/a \rceil^n$ for a positive chain dominated by a.
C2	Exponential decoherence damping and its unitary limit.
C3	Dimensional decoherence estimate against the CPLEAR/KLOE bound.
C4	EHNS semipositivity ($\alpha, \gamma \geq 0$, $\alpha\gamma \geq \beta^2$) is necessary but not sufficient for complete positivity.
C5	Margolus-Levitin orthogonalization rate vanishes at the ground state while proper time still flows.
C6	Amplitude-blind selection matches Born only on the degenerate locus.
C7	Amplitude-blind selection is not Born-universal for $n \geq 2$.
C8	Energy-neutrality of the dephasing channel $\mathcal{D}$ specifically: $\mathrm{tr}(H \cdot \mathcal{D}(\rho)) = \mathrm{tr}(H \cdot \rho)$ when $[H, P_k] = 0$ for all pointer projectors P_k, with failure when $[H, P_k] \neq 0$; the commuting form is sufficient for $\mathcal{D}$ alone and does not extend to a general quiescent map (cited in Theorem 11(b) and Appendix A).
T1, T1′	$\mathcal{C}_1$ witness rate and its operator-level construction.
T2	$\mathcal{C}_2$ witness rate.
T3	$\mathcal{C}_3$ rate $\delta_3 = n \sum_k p_k^2 - 1$: nonnegativity, the upper bound $n - 1$, saturation at a point mass, and vanishing at the uniform distribution.
T4	$\mathcal{C}_4$ Gisin-Polchinski rate $\varepsilon_4 = \lambda \lvert \mathrm{tr}\, \rho_A^2 - \mathrm{tr}\, \rho_A'^2 \rvert$.
T5	$\mathcal{C}_4^{\mathrm{sd}}$ state-dependent re-description: per-fiber linearity, simplex match of $\varepsilon_4^{\mathrm{sd}}$, and output identity with $\mathcal{C}_4$.
T6	$M_{\phi'}$ doubly-stochastic mixing companion at the matched angle.
T7	General no-signaling cross-term identity at arbitrary pointer multiplicity, arbitrary bipartite dimension, and complex record overlaps (cited from Proposition 33).
T8	κ-asymmetry separating $\mathcal{C}_2$ from $M_{\phi'}$, for Proposition 29.

Remark 38 (Status). The codes fall into four classes, and the class governs what the code licenses.

Machine-checked (F1–F5, F7, F9; C1, C4, C6–C8; T3; T7): a Lean 4 theorem against mathlib establishes the stated identity, with no axioms beyond the mathlib standard three and no incomplete proofs. Table 4 names the file and theorem for each. These artifacts are quantified

over all dimensions and all parameters, so they are strictly stronger than the fixed-dimension instances they index.

Partially machine-checked (F6): the state-level content, that the post-selection and no-selection states coincide on the pointer marginal, is machine-checked; the step from state equality to optimal discrimination probability 1/2 is the Helstrom bound, cited rather than re-proved.

Not formalized (T1, T1′, T2, T6, T8): the $\mathcal{C}_1$ and $\mathcal{C}_2$ rates and the diagnostics computed from them are trigonometric identities on a two-dimensional pointer space, established in the body. They carry no Lean artifact, for the same reason as T4 and T5: the trace-norm and χ^2 estimates they rest on have no counterpart in the formal layer.

Not propositions (F8; C2, C3, C5): a rate quoted from the spontaneous-localization literature and checked for sign, an exponential-damping solution and its unitary limit, a dimensional estimate compared against an experimental bound, and a numerical evaluation of the Margolus-Levitin rate at the ground state. These are calculations and comparisons, not theorems about the quiescent class, and they carry no verification artifact. The remaining tightness rates (T4, T5) rest on trace-norm and purity-contrast estimates for which the formalization has no mathlib support; they are stated in the body and carry no artifact here.

Verification of a claim here does not upgrade that claim's status beyond what the body proof establishes. The artifacts check the algebraic kernels and witness identities; they do not replace the modeling assumptions of Definition 5.

Artifact index. The artifacts are deposited as flat Lean 4 source files in the supplementary archive, built against mathlib at toolchain `leanprover/lean4:v4.29.1`. Each file compiles standalone.

Table 4: Formal artifact index.

Code	Lean file and theorem	Scope
F1	`PresentTenseForcing.lean`: `mulVec_fix_all_imp_one`, `mulVec_fix_vertices_imp_one`	all n
F2	`PresentTenseForcing.lean`: `born_universality_necessary`	$n = 2$ witness
F3	`BITDephasing.lean`: `f9_expectation_scalar_block`; `f9_operator_identity` (rank-1 pointer projectors, operator-level strengthening)	all dimensions
F4	`BITDephasing.lean`: `loud_horn_not_covariant`, `loud_horn_injects`, `loud_horn_populations`	qubit witness
F5	`PresentTenseForcing.lean`: `mulVec_fix_simplex_imp_one`, `affine_fix_vertices_fix_simplex`	all n
F6	`BITDephasing.lean`: `f9_operator_identity`, `f9_silence` (state level); Helstrom step cited	all dimensions
F7	`PresentTenseNoSignaling.lean`: `ptraceA_dephase_eq`, `ptraceA_round_eq`, `ptraceA_foldl_eq`, `ptraceA_swap_eq`	all (m, n)

(continued on next page)

(Table 4, continued)

Code	Lean file and theorem	Scope
F8	no artifact (rate comparison, not a proposition)	
F9	`BITDephasing.lean`: `f9_expectation_scalar_block` ($\mathcal{A}_P$-state equality from block-weight agreement alone); `f9_operator_identity`, `f9_silence` (rank-1 pointer projectors, full operator equality under pointer covariance)	all dimensions
C1	`BITCounting.lean`: `fiber_count_bound`	all n
C2, C3, C5	no artifact (calculation or comparison, not a proposition)	
C4	`BITCounting.lean`: `ehns_not_psd`	explicit witness
C6	`BITCounting.lean`: `blind_eq_born_iff`	qubit
C7	`BITCounting.lean`: `blind_not_born_universal`	all $n \geq 2$
C8	`BITDephasing.lean`: `trace_dephase_eq`, `trace_dephase_ne_of_noncommuting`	all dimensions
T1, T1′, T2, T6, T8	no artifact (trigonometric rate identities; established in the body)	
T3	`BITCounting.lean`: `delta3_nonneg`, `delta3_le`, `delta3_point_mass`, `delta3_uniform`	all n
T4, T5	no artifact (trace-norm and purity-contrast estimates)	
T7	`PresentTenseNoSignaling.lean`: `ptraceA_dephase_eq`, `dephase_offdiag_zero`	all (m, n), complex entries

C Limits of Operational Discrimination

$(\beta) \not\Rightarrow (\alpha)$: a p-indexed kernel outside the SelFact type

A per run kernel can read the realized amplitudes (violating (α)) while its run-ensemble average still equals $\mathcal{D}_A^\theta$ (satisfying (β)). The object below inhabits the broader class of p-indexed stochastic kernels (those with a p-port), which lies outside the SelFact type of Definition 5(i); it is recorded here to illustrate that the SelFact typing is strictly stronger than the ensemble level silence condition.

Explicit construction: let $\Lambda = [0, 1]$ with Lebesgue measure, and set

$$K(1 \mid p, \lambda) \;=\; p_1 + \delta\, p_1(1 - p_1)\sin(2\pi\lambda), \qquad K(2 \mid p, \lambda) = 1 - K(1 \mid p, \lambda), \qquad 0 < \delta \leq 1.$$

Since $0 \leq p_1 - \delta\, p_1(1 - p_1) \leq K(1 \mid p, \lambda) \leq p_1 + \delta\, p_1(1 - p_1) \leq 1$ for $0 < \delta \leq 1$, this is a valid stochastic kernel. This kernel reads p_1 at every run ((α) fails) but satisfies $\mathbb{E}_\lambda[K(1 \mid p, \cdot)] = p_1$ for all p (since $\sin(2\pi\lambda)$ integrates to zero over $[0, 1]$), so (β) holds. The construction generalises: any zero-mean λ-modulation of a p-dependent perturbation preserves (β) while inhabiting the broader class of p-port kernels.

Underdetermination of the R_4 description (proof of Proposition 27)

$\mathcal{C}_4$ and $\mathcal{C}_4^{\mathrm{sd}}$ are formula identical on every input (T5(c) in Appendix B): the composite function $\rho \mapsto M_{\mathrm{tr}(\rho^2)}(\rho)$ equals $\mathcal{C}_4(\rho)$ for all ρ. Consequently no output state measurement on any single input separates them, and no multi-input protocol comparing per-input outputs can distinguish them, because the outputs are identical at each input. The purity-contrast rate $\varepsilon_4 = \lambda|r_1^2 - r_2^2|$ is accessible by reading the scalar $\mathrm{tr}(\rho^2)$; this scalar access can be described either as R_4 violation (ensemble nonlinearity through a quadratic functional of ρ) or as aggregate state-dependence at the ensemble level (the state-dependent redescription of $\mathcal{C}_4$ discussed in the $\mathcal{C}_4$ companion paragraph of Section 4). The rate does not discriminate which description applies; both describe the same map.

The $\mathcal{C}_4$ partial witness is accordingly the strongest honest R_4 boundary statement available: $\varepsilon_4 > 0$ witnesses the relaxed set $\{R_1, R_3, R_4\}$ rather than R_4 alone, so strict sense (1) does not hold; sense (2) is obstructed by the output identity of T5(c), and no output-level or rate-level side condition removes that obstruction.

D Experimental bounds landscape

The Lindblad Flow exit model (Section 8) with coupling γ is constrained by several independent experimental channels. Numerical leadership among CSL bounds changes over time and varies with model convention, so this appendix identifies falsifier channels instead of maintaining a live global leaderboard. The landscape below is a curated snapshot of cited bounds, and the tightest bound in each channel at any given time sets the Flow exit viability window.

Convention. Bounds are stated at the canonical nucleon mass scale ($m_n \approx 1.67 \times 10^{-27}$ kg, $A = 1$) with correlation length $r_C = 10^{-7}$ m. The conversion from the CSL localization rate λ_{CSL} to the Lindblad coupling γ is fixed by

$$\gamma \;=\; \frac{\lambda_{\mathrm{CSL}}}{r_C^2}, \qquad r_C = 10^{-7}\,\mathrm{m}, \tag{6}$$

where λ_{CSL} has units s^{-1} and γ has units $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$. For a composite body of nucleon number A with collective position coupling, the heating rate is enhanced by A^2 under this convention.

Representative cited bounds.

- **Spontaneous X-ray, xenon (XENON Collaboration 2026 [30]).** The leading emission-channel constraint. Low-energy electronic-recoil data over 1–140 keV, with a spectral model accounting for cancellation between opposing electron and proton charges in xenon, set world-leading limits on the Markovian CSL and Diosi-Penrose parameters, improving the previous best by two orders of magnitude and a factor of five respectively. No X-ray excess is observed.
- **Spontaneous X-ray, germanium (Donadi et al. 2021 [13]).** An underground germanium measurement of spontaneous radiation. Its result is specific to the gravity-related Diosi-Penrose model, whose natural parameter-free form it excludes; it is not a universal CSL-to-γ bound, and it is quoted here for the DP channel only.
- **Free-falling test masses (Carlesso et al. 2016 [9]; rotational noise, 2025 [8]).** Macroscopic mechanical force-noise channel across LIGO, LISA Pathfinder, and AURIGA. The strongest bound in the 2016 analysis is set by LISA Pathfinder rather than by LIGO, and the LISA Pathfinder constraint is further improved by the 2025 rotational-noise analysis. Next-generation gravitational-wave detectors (Einstein Telescope, Cosmic Explorer, projected 2030s) are expected to extend the mechanical channel further.

- **Cosmological and astrophysical heating (Adler 2007 [2]; Fixsen et al. 1996 [16]).** Independent heating channel, in which a universal position-coupling dissipator deposits energy in diffuse matter and radiation. Adler sets upper and lower bounds on the CSL parameters from latent image formation and from heating of the intergalactic medium; Fixsen et al. measure the CMB spectrum over the full COBE FIRAS data set and bound its departure from a blackbody. Neither is a bound on γ under (6), and the paper derives none: the channel is recorded here for its independence from the laboratory settings above.

- **Joint heating + spontaneous emission (ratio).** A joint constraint rather than a new bound: the ratio of the heating-derived γ (mechanical force noise, [8, 9]) to the emission-derived γ (xenon X-ray, [30]) is a fixed Lindblad Flow exit prediction at each mass scale under (6). The two channels it draws on are the ones listed above; the paper states the ratio as a prediction and assigns it no numerical value. See Remark 36.

Status. This appendix establishes two things and no third: the conversion convention (6), and which channels bound the Flow exit window $0 < \gamma < \gamma_{\max}$. The upper end of that window is set by the leading emission channel, currently xenon [30], with the mechanical channel bounded by LISA Pathfinder [8, 9] and expected to extend further with next-generation gravitational-wave detectors (ET/CE). The numerical value of $\gamma_{\max}$ follows from the leading limit through (6) and moves with it; take it from the cited experiments, not from a figure fixed here.

What these experiments do and do not test. Each channel constrains a model-specific parameter through a model-specific observable: λ_{CSL} and r_C for CSL, the regularization length for Diosi-Penrose, and γ for the Lindblad Flow exit under (6). None of them measures the Helstrom quantity of Theorem 11, which is a statement about $\mathcal{A}_P$ discrimination and is saturated at $1/2$ by theorem within the quiescent class; the two kinds of claim are distinguished in Remark 17. The relation is one-directional: a confirmed, model-attributed signal in any of these channels would falsify the unitary-Flow extension under test. It would not by itself show that branch selection caused it, nor that a silent actuality augmentation is absent; a null result constrains that one exit and leaves the theorem untouched.